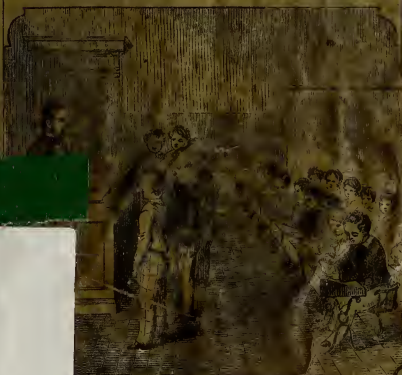


DEAN'S
INTELLECTUAL
ARITHMETIC.



PUBLISHED BY
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THE
INTELLECTUAL
ARITHMETIC:

IN WHICH

THE FUNDAMENTAL IDEAS OF THE SCIENCE ARE INDUCTIVELY
TAUGHT, AND THE SOLUTIONS OF QUESTIONS ARE
GIVEN IN THE FORM OF COMPLETE AND
LOGICAL ANALYSES;

Illustrated by Numerous Examples.

DESIGNED FOR

CLASS RECITATION AND EXERCISE IN PUBLIC SCHOOLS
AND ACADEMIES.

BY

PHILOTUS DEAN, A.M.

PITTSBURGH:

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PREFACE.

MENTAL or Intellectual, as distinguished from Written, Arithmetic, implies the performance of the computations necessary to the solution of Mathematical questions without the assistance of written operations or visible signs. Its importance arises from the fact that business life requires quickness of insight into its practical problems, readiness and tact in dealing with them, familiarity with their principles, and rapidity in reaching their conclusions. While it is the province of Mental Arithmetic to satisfy these requirements, it is also a most efficient means of educating the intellectual powers, more particularly those employed in logical reasoning and methodical statement. It is a valuable auxiliary to Written Arithmetic; and the skill given by its analyses will often remain, when the memory no longer retains burdensome rules and formal solutions. In fact, he who is thoroughly versed in the analytical processes of Mental Arithmetic, stands in need of but a few rules for computation. Another important effect of its proper pursuit is to improve the power of the pupil to utter his reasonings in a full, clear, methodical and forcible style. Most pupils fail in their powers of expression, even when they comprehend the steps of a solution. In this, as in physical exercise, the repetition of models and forms of analysis imparts the skill of habit.

This work is intended to lead the learner, by progressive development, from the simplest fundamental principles of the science to such of its higher and more involved applications as fall properly within the scope of Mental Arithmetic. He is required to *think* his way as he advances, and is furnished with such models for expressing his thoughts as experience has shown to be best. Although the work is so constructed that the pupil can begin his arithmetical studies with it, yet it is advisable to go first through the Primary Arithmetic, because his mind will then have acquired that familiarity with the subject which will make his progress in this more rapid and effectual.

SUGGESTIONS TO TEACHERS.

THE portion to be recited should be assigned to the class a sufficient time before recitation to let it be thoroughly acquired. Let only so much be assigned as can be well prepared, whether it be the whole or only a part of a lesson. Long tasks weary and dispirit, and are less perfectly performed, while the same labor, subdivided, will be accomplished with more alacrity and profit.

The pupil should be taught to prepare, not only the correct results of the computations, but an analysis for each question so perfectly as to preclude failure in the recitation.

It is by many considered best to allow no book in recitation except to the teacher, who is to pronounce slowly and distinctly the question to be solved, after which the pupil is to repeat the statement of the question, and then give the analysis. This should be the common custom; but, in long questions in the more advanced parts of the book, it will be better to dispense with the formality of repeating or reproducing the question after the teacher, and to proceed at once to the analysis.

In these oral performances, care should be taken by the teacher to secure clear and distinct utterance, correct pronunciation, intonations and inflections of voice, and grammatical sentences. The exercise thus becomes not only a drill in mental computation and logical reasoning, but a valuable lesson in Grammar and Rhetoric, or in the art of speaking with correctness and force.

It creates lively interest and close attention on the part of the pupils, to allow them to express their criticisms on each performance, if they signify, by lifting the hand, that they have any to offer.

INTELLECTUAL ARITHMETIC.

CHAPTER I.

EXERCISES IN COUNTING AND NOTATION.

LESSON I.

1. A single tree is how many? ANSWER. One tree.
2. One tree and one tree are how many trees?
3. Two trees and one tree are how many trees?
4. Three trees and one tree are how many trees?
5. Four boys and one boy are how many boys?
6. Five boys and one boy are how many boys?
7. Six boys and one boy are how many boys?
8. Seven girls and one girl are how many girls?
9. Eight girls and one girl are how many girls?
10. Nine girls and one girl are how many girls?
11. Ten men and one man are how many men?
12. Eleven men and one man are how many?
13. Twelve men and one man are how many?
14. Thirteen cents and one cent are how many?
15. Fourteen cents and one cent are how many?
16. Fifteen cents and one cent are how many?
17. Sixteen plums and one plum are how many?
18. Seventeen plums and one plum are how many?
19. Eighteen plums and one plum are how many?

20. Nineteen pears and one pear are how many?
21. Twenty pears and one pear are how many?
22. Twenty-one pears and one pear are how many?
23. Begin at the thumb of your left hand, call it *twenty-one*, and count the fingers of both hands.
24. Begin at the same thumb, call it *thirty-one*, and count the fingers of both hands.
25. Call the thumb *forty-one*, and count as before.
26. Call the thumb *fifty-one*, and count as before.
27. Call the thumb *sixty-one*, and count as before.
28. Call the thumb *seventy-one*, and count as before.
29. Call the thumb *eighty-one*, and count as before.
30. Call the thumb *ninety-one*, and count as before.
31. Mention in order all the numbers which have just been counted, whose names end with the letters *ty*.
32. With what names, put after the words *twenty*, *thirty*, *forty*, *fifty*, *sixty*, *seventy*, *eighty*, *ninety*, and *one hundred*, do you make the names of the next higher numbers?
33. Begin at *one hundred and ten*, and count ten.
34. Begin at *one hundred and twenty*, and count ten.
35. Begin at *one hundred and thirty*, and count ten.
36. Begin at *one hundred and forty*, and count ten.
37. Begin at *one hundred and fifty*, and count ten.
38. Begin at *one hundred and sixty*, and count
39. Begin at *one hundred and seventy*, and count ten.
40. Begin at *one hundred and eighty*, and count ten.
41. Begin at *one hundred and ninety*, and count ten.
42. Begin at *two hundred and ninety*, and count ten.
43. Begin at *three hundred and ninety*, and count ten.
44. Begin at *four hundred and ninety*, and count ten.
45. Begin at *five hundred and ninety*, and count ten.
46. Begin at *six hundred and ninety*, and count ten.
47. Begin at *seven hundred and ninety*, and count ten.
48. Begin at *eight hundred and ninety*, and count ten.
49. Begin at *nine hundred and ninety*, and count ten.
50. Mention in order all the *hundreds* which have just been named. How many are they?

LESSON II.

1. What are the characters called, which are generally used to represent numbers?

ANS. They are called *figures*.

2. How many and what are the figures?

ANS. They are ten, and are written thus:—

0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
naught, one, two, three, four, five, six, seven, eight, nine.

3. How are numbers greater than *nine* represented?

ANS. By using more than one of the figures.

4. How is *ten* represented?

ANS. The figure 1 represents *ten*, if it is written as the second figure toward the left; thus:—

1 with 0 (10) means *ten* and *naught*, which is *ten*.

1 with 1 (11) means *ten* and *one*, or *eleven*.

1 with 2 (12) means *ten* and *two*, or *twelve*.

1 with 3 (13) means *ten* and *three* or *thirteen*.

1 with 4 (14) means *ten* and *four*, or *fourteen*.

1 with 5 (15) means *ten* and *five*, or *fifteen*.

1 with 6 (16) means *ten* and *six*, or *sixteen*.

1 with 7 (17) means *ten* and *seven*, or *seventeen*.

1 with 8 (18) means *ten* and *eight*, or *eighteen*.

1 with 9 (19) means *ten* and *nine*, or *nineteen*.

5. How is *twenty* represented?

ANS. The figure 2 represents *twenty*, if it is written as the second figure toward the left, thus:—

2 with 0 (20) means *twenty* and *naught*, or *twenty*.

2 with 1 (21) means *twenty* and *one*, or *twenty-one*.

6. What does 2 followed by 2 mean? By 3? By 4?
By 5? By 6? By 7? By 8? By 9?

7. How is *thirty* represented?

ANS. The figure 3 represents *thirty*, if it is written as the second figure toward the left, thus:—

3 with 0 (30) means *thirty* and *naught*, or *thirty*.

8. What is the meaning of the second figure toward the left and of the whole number in 40? 50? 60? 70? 80? 90?

31	38	46	54	62	69	77	85	93
32	39	47	55	63	71	78	86	94
33	41	48	56	64	72	79	87	95
34	42	49	57	65	73	81	88	96
35	43	51	58	66	74	82	89	97
36	44	52	59	67	75	83	91	98
37	45	53	61	68	76	84	92	99

9. How are *hundreds* represented?

Ans. By the third figure toward the left; thus:—

1 with two 0's (100) means 1 *hundred*, and *naught*.

2 with 10 (210) means 2 *hundred*, and *ten*.

3 with 21 (321) means 3 *hundred*, and *twenty-one*.

10. What is the meaning of the third figure toward the left, and of the whole number, in 200? 300? 131? 142? 254? 265? 376? 387? 498? 422? 533? 655? 766? 777? 899? 999? 904? 409?

11. How are *thousands* represented?

Ans. By the fourth figure toward the left.

12. Explain 1000; 2000; 3100; 4200; 5430; 6540; 7654; 8765; 9006; 8017; 7603; 6201; 9875.

13. What do the two I's, at the head of this lesson mean?

Ans. They are Roman figures, meaning *two*. They used I for *one*, V *five*, X *ten*, L *fifty*, C *one hundred*, D *five hundred*, and M *one thousand*.

TABLE.

I	is	1	XI	is	11	XXI	is	21	CCC	is	300
II	is	2	XII	is	12	XXX	is	30	{ CCCC	is	400
III	is	3	XIII	is	13	XL	is	40	{ or CD		
IV	is	4	XIV	is	14	L	is	50	D	is	500
V	is	5	XV	is	15	LX	is	60	DC	is	600
VI	is	6	XVI	is	16	LXX	is	70	DCC	is	700
VII	is	7	XVII	is	17	LXXX	is	80	DCCC	is	800
VIII	is	8	XVIII	is	18	XC	is	90	DCCCC	is	900
IX	is	9	XIX	is	19	C	is	100	M	is	1000
X	is	10	XX	is	20	CC	is	200	MM	is	2000

CHAPTER II.

EXERCISES IN ADDITION.

LESSON I.

1. How many are 2 books and 1 book?

ANS. Two books and one book are three books.

2. If I ask, "How many are 2 and 1?" without mentioning any particular things, what do I mean?

ANS. You mean to ask how many are 2 single things of *any* kind, and 1 more of the same kind.

3. Does the *kind* of things affect their *number*?

ANS. It does not; therefore we can learn numbers without reference to existing things.

4. What are numbers called which signify things of a certain kind, as 2 *books*, 3 *men*, &c.?

ANS. Such numbers are called *concrete* numbers.

5. What are numbers called which do not signify things of a definite kind, as 2, 3, 5, &c.?

ANS. Such numbers are called *abstract* numbers.

6. How many are 2 and 2?

MODEL OF ANSWER.—2 and 2 are 4.

7. How many are 2 and 3? 2 and 4? 2 and 5? 2 and 6? 2 and 7? 2 and 8? 2 and 9? 2 and 10?

8. Ada spent 2 cents at one store and 3 at another; how many cents did she spend at both stores?

SOLUTION.—Since 2 and 3 are five, she spent 5 cents.

9. Ann's father sold 2 sheep to one man and 4 to another; how many did he sell to both?

10. Are 2 and 4 the same amount as 4 and 2?

11. John had 5 cents, and gained 2 cents more by trading; how many cents had he then?

12. Jane bought candy for 2 cents, and nuts for 6 cents; how much did she give for both?

13. Two boys were at play, and 7 more came; how many were there then?

14. Eight girls were at the swing, and 2 more came; how many were there then?

15. Mary had 8 pins on the cushion, and she put 2 more on it; how many were on it then?

16. If you give 9 cents for a copy-book, and 2 cents for a pen, how many cents do you give for both?

17. George caught 3 squirrels, and Peter 3; how many squirrels did both catch?

18. In a class were 4 girls and 3 boys; how many pupils were in the class?

19. In a party were 4 men and 4 women; how many persons were in the party?

20. One day Frank walked 5 miles, and the next day 3; how many miles did he walk in both days?

21. Herman caught 6 fishes in the stream, and 3 in the pond; how many did he catch in both places?

22. I bought 8 loads of coal of one man, and 3 of another; how many loads did I buy of both men?

23. One day Carrie wrote 7 lines in her copy-book, and the next day 3; how many in both days?

24. In a walk Susan saw 9 robins, and 3 finches; how many birds were these?

25. Horace's trap caught 5 rats, and Edward's 4; how many rats did both catch?

26. Charles sold his ball for 7 cents, and his top for 4 cents; how many cents did he get for both?

27. The dining-room has 6 chairs, and the kitchen 4; how many chairs have both rooms?

28. Gave 9 cents for a slate, and 4 for a sponge; how many cents did I give for both?

29. In one pasture are 8 cows, and in another 4; how many cows are there in both pastures?

LESSON II.

1. When we say "2 and 4 are 6," what are we said to do with the 2 and 4?

ANS. We are said to *add* them together, which means that we unite them into one number, making 6.

2. What is the 6 called in that case?

ANS. It is called the *sum*, or *amount* of 2 and 4.

3. David hoed 5 rows of corn one day, and 5 the next; how many rows did he hoe in both days?

4. A drug-store had 5 bottles in one window, and 6 in the other; how many in both?

5. A hail-storm broke 5 panes in one window, and 7 in another; how many in both?

6. Sheldon earned 5 cents one day and 9 another day; how many in both days?

7. In an orchard are 8 apple-trees and 5 peach-trees; how many trees are in the orchard?

8. How many pieces make a set of 6 tea-cups and saucers?

9. If 6 play against 7 in a game of ball, how many are playing?

10. Lucy gave 6 cents to a blind man, and 8 cents to a poor woman; what was the sum of her gifts?

11. One hen had 6 chickens, and another 9; how many chickens had both hens?

12. A man bought 7 dollars' worth of sugar, and 8 dollars' worth of flour; what amount did he pay for both?

13. A farmer sold potatoes for 7 dollars, and corn for 7 dollars; what sum did he receive for both?

14. After John had eaten 7 pears he had 9 remaining; how many had he at first?

15. A farmer plowed 8 acres one week, and 9 the next; how many did he plow in both weeks?

16. Thomas had 8 cents in his "savings-box," and put in 8 more; what sum was in it then?

17. A hunter shot 9 deer, and 9 turkeys; of how many animals did his game consist?

18. How many are 3 and 2? 4 and 2? 5 and 2? 6 and 2? 7 and 2? 8 and 2? 9 and 2?

19. How many are 3 and 4? 4 and 3? 3 and 5? 5 and 3? 3 and 6? 6 and 3? 3 and 7? 7 and 3? 3 and 8? 8 and 3? 3 and 9? 9 and 3?

20. How many are 4 and 5? 5 and 4? 4 and 6? 6 and 4? 4 and 7? 7 and 4? 4 and 8? 8 and 4? 4 and 9? 9 and 4?

21. How many are 5 and 6? 6 and 5? 5 and 7? 7 and 5? 5 and 8? 8 and 5? 5 and 9? 9 and 5?

22. How many are 6 and 7? 7 and 6? 6 and 8? 8 and 6? 6 and 9? 9 and 6?

23. How many are 7 and 8? 8 and 7? 7 and 9?

24. How many are 9 and 7? 8 and 9? 9 and 8?

25. Franklin spent 10 days at his uncle's, and was 1 day going there, and 1 day returning; how long was he gone from home?

26. His father gave him 10 cents, and his uncle 3 cents; how much did both give him?

27. There were 10 persons in a car, and 5 more entered it; how many were then in the car?

28. In one part of the town, 10 houses were burned, and 4 in another part; how many in all?

29. A man's fence needed 10 new rails at one spot, and 7 at another; how many at both places?

30. It rained 10 times in May, and 6 times in June; how many times did it rain in both months?

31. The lower stairs have 10 steps, and the upper 9; how many steps have both stairs?

32. Gave 10 dollars for a coat, and 8 dollars for vest and pants; how much did the suit cost?

33. If you have a dime, which is 10 cents, and earn another dime, how many cents have you then?

34. How many are 2 and 10? 10 and 2? 3 and 10? 10 and 3? 4 and 10? 10 and 4? 5 and 10? 10 and 5? 6 and 10? 10 and 6? 7 and 10? 10 and 7? 8 and 10? 10 and 8? 9 and 10? 10 and 9? 10 and 10?

LESSON III.

1. If I ask "How many are 11 *plus* 2?" what do I mean by the word *plus*?

ANS. The word *plus* means *more*, and you ask how many are 11 *made more*, that is, *increased* by 2. The word *plus* denotes the addition of the number after it to the number before it.

2. How many are 11 plus 1?

MODEL OF ANSWER.—11 plus 1 are 12.

3. How many are 11 plus 3? 11 plus 2? 11 plus 4?
4. How many are 11 plus 6? 11 plus 5? 11 plus 7?
5. How many are 11 plus 9? 11 plus 8? 11 plus 10?
6. How many are 12 plus 3? 12 plus 2? 12 plus 4?
7. How many are 12 plus 6? 12 plus 5? 12 plus 7?
8. How many are 12 plus 9? 12 plus 8? 12 plus 10?
9. How many are 13 plus 3? 13 plus 2? 13 plus 4?
10. How many are 13 plus 6? 13 plus 5? 13 plus 7?
11. How many are 13 plus 9? 13 plus 8? 13 plus 10?
12. If I *write* the question "How many are $4 + 2$?" what is meant by that sign between the 4 and 2?

ANS. It is the sign for *plus*, and the question is read, "How many are 4 *plus* 2?"

13. How many are $14 + 3$? $14 + 2$? $14 + 4$?
14. How many are $14 + 6$? $14 + 5$? $14 + 7$?
15. How many are $14 + 9$? $14 + 8$? $14 + 10$?
16. How many are $15 + 3$? $15 + 2$? $15 + 4$?
17. How many are $15 + 6$? $15 + 5$? $15 + 7$?
18. How many are $15 + 9$? $15 + 8$? $15 + 10$?
19. How many are $16 + 3$? $16 + 2$? $16 + 4$?
20. How many are $16 + 6$? $16 + 5$? $16 + 7$?
21. How many are $16 + 9$? $16 + 8$? $16 + 10$?
22. How many are $17 + 3$? $17 + 2$? $17 + 4$?
23. How many are $17 + 6$? $17 + 5$? $17 + 7$?
24. How many are $17 + 9$? $17 + 8$? $17 + 10$?
25. How many are $18 + 3$? $18 + 2$? $18 + 4$?
26. How many are $18 + 6$? $18 + 5$? $18 + 7$?

27. How many are $18 + 9$? $18 + 8$? $18 + 10$?
28. How many are $19 + 3$? $19 + 2$? $19 + 4$?
29. How many are $19 + 6$? $19 + 5$? $19 + 7$?
30. How many are $19 + 9$? $19 + 8$? $19 + 10$?
31. If you buy 10 pounds of sugar and 5 of coffee, how many pounds of groceries have you purchased?
32. A man cut 11 tons of hay at the first mowing, and 7 at the second; how much was his hay crop?
33. He sold one load of hay for 12 dollars, and another for 9; how much did he receive for both?
34. His wife sold 13 pounds of butter one week, and 8 the next; how many in both weeks?
35. His son John sold 15 dozen eggs one month, and 6 dozen the next; how many dozen in both months?
36. His daughter Susan sold 16 quarts of milk to one neighbor, and 9 to another; how many to both?
37. If you owe Stephen 17 cents, and Alfred 5 cents, how much are you in debt to both?
38. If Philip owes you 18 cents, and William 7 cents, how much do both owe you?
39. Louis is 19 years old; how old will he be at the end of 2 years more, if he lives?
40. John is 10 years old; how old will he be, if he lives 10 years more?
41. Samuel is 12 years old; how old will he be, if he lives 8 years more?
42. His sister is 8 years old; how old will she be, if she lives 11 years more?
43. A merchant took in 5 dollars in the morning, and 13 in the afternoon; how much in the whole day?
44. He bought at one time 8 hundred dollars' worth of goods, at another 16 hundred dollars' worth; what was the amount of both purchases?
45. A king having 15 thousand men in his army, added to it 9 thousand more; how large was his army then?
46. A man, worth 14 thousand dollars, gained by a speculation 6 thousand dollars more; how many thousand dollars was he worth then?

LESSON IV.

1. Add 2 to 0, 10, 20, 30, 40, 50, 60, 70, 80, 90.

MODEL OF ANSWER.—2 and 0 are 2; 2 and 10 are 12; 2 and 20 are 22; 2 and 30 are 32; 2 and 40 are 42; 2 and 50 are 52; 2 and 60 are 62; 2 and 70 are 72; 2 and 80 are 82; 2 and 90 are 92.

2. Add 2 to 1, 11, 21, 31, 41, 51, 61, 71, 81, 91.
3. Add 2 to 2, 12, 22, 32, 42, 52, 62, 72, 82, 92.
4. Add 2 to 3, 13, 23, 33, 43, 53, 63, 73, 83, 93.
5. Add 2 to 4, 14, 24, 34, 44, 54, 64, 74, 84, 94.
6. Add 2 to 5, 15, 25, 35, 45, 55, 65, 75, 85, 95.
7. Add 2 to 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.
8. Add 2 to 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.
9. Add 2 to 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
10. Add 2 to 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
11. Add 3 to 0, 10, 20, 30, 40, 50, 60, 70, 80, 90.
12. Add 3 to 1, 11, 21, 31, 41, 51, 61, 71, 81, 91.
13. Add 3 to 2, 12, 22, 32, 42, 52, 62, 72, 82, 92.
14. Add 3 to 3, 13, 23, 33, 43, 53, 63, 73, 83, 93.
15. Add 3 to 4, 14, 24, 34, 44, 54, 64, 74, 84, 94.
16. Add 3 to 5, 15, 25, 35, 45, 55, 65, 75, 85, 95.
17. Add 3 to 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.
18. Add 3 to 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.
19. Add 3 to 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
20. Add 3 to 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
21. Add 4 to 0, 10, 20, 30, 40, 50, 60, 70, 80, 90.
22. Add 4 to 1, 11, 21, 31, 41, 51, 61, 71, 81, 91.
23. Add 4 to 2, 12, 22, 32, 42, 52, 62, 72, 82, 92.
24. Add 4 to 3, 13, 23, 33, 43, 53, 63, 73, 83, 93.
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50. Add 6 to 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
51. Add 7 to 0, 10, 20, 30, 40, 50, 60, 70, 80, 90.
52. Add 7 to 1, 11, 21, 31, 41, 51, 61, 71, 81, 91.
53. Add 7 to 2, 12, 22, 32, 42, 52, 62, 72, 82, 92.
54. Add 7 to 3, 13, 23, 33, 43, 53, 63, 73, 83, 93.
55. Add 7 to 4, 14, 24, 34, 44, 54, 64, 74, 84, 94.
56. Add 7 to 5, 15, 25, 35, 45, 55, 65, 75, 85, 95.
57. Add 7 to 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.
58. Add 7 to 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.
59. Add 7 to 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
60. Add 7 to 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
61. Add 8 to 0, 10, 20, 30, 40, 50, 60, 70, 80, 90.
62. Add 8 to 1, 11, 21, 31, 41, 51, 61, 71, 81, 91.
63. Add 8 to 2, 12, 22, 32, 42, 52, 62, 72, 82, 92.
64. Add 8 to 3, 13, 23, 33, 43, 53, 63, 73, 83, 93.
65. Add 8 to 4, 14, 24, 34, 44, 54, 64, 74, 84, 94.
66. Add 8 to 5, 15, 25, 35, 45, 55, 65, 75, 85, 95.
67. Add 8 to 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.
68. Add 8 to 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.
69. Add 8 to 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
70. Add 8 to 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.

LESSON V.

1. Add 9 to 0, 10, 20, 30, 40, 50, 60, 70, 80, 90.
2. Add 9 to 1, 11, 21, 31, 41, 51, 61, 71, 81, 91.
3. Add 9 to 2, 12, 22, 32, 42, 52, 62, 72, 82, 92.
4. Add 9 to 3, 13, 23, 33, 43, 53, 63, 73, 83, 93.
5. Add 9 to 4, 14, 24, 34, 44, 54, 64, 74, 84, 94.
6. Add 9 to 5, 15, 25, 35, 45, 55, 65, 75, 85, 95.
7. Add 9 to 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.
8. Add 9 to 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.
9. Add 9 to 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
10. Add 9 to 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
11. How many are 1 and 2 and 3?

MODEL OF ANSWER.—1 and 2 are 3, and 3 more are 6.

12. How many are 2 and 3 and 4? 1 and 3 and 4?
13. How many are 1 and 3 and 5? 2 and 3 and 5?
14. How many are 2 and 2 and 3? 2 and 2 and 4?
15. How many are 2 and 1 and 4? 2 and 4 and 5?
16. How many are 2 and 5 and 1? 2 and 4 and 2?
17. How many are 2 and 3 and 3? 2 and 2 and 5?
18. How many are 2 and 3 and 6? 2 and 6 and 2?
19. How many are 3 and 1 and 7? 3 and 2 and 7?
20. How many are 3 and 4 and 5? 3 and 5 and 2?
21. How many are 3 and 5 and 1? 3 and 6 and 1?
22. How many are 3 and 2 and 6? 3 and 3 and 3?
23. How many are 3 and 4 and 3? 3 and 4 and 4?
24. How many are 3 and 5 and 5? 3 and 1 and 7?
25. How many are 3 and 2 and 4? 3 and 5 and 6?
26. How many are 4 and 2 and 1? 4 and 2 and 2?
27. How many are 4 and 3 and 3? 4 and 4 and 3?
28. How many are 4 and 4 and 4? 4 and 5 and 4?
29. How many are 4 and 5 and 5? 4 and 5 and 6?
30. How many are 4 and 6 and 6? 4 and 6 and 7?
31. How many are 4 and 5 and 7? 4 and 7 and 7?
32. How many are 4 and 6 and 8? 4 and 7 and 8?
33. How many are 4 and 8 and 8? 4 and 7 and 9?
34. How many are 4 and 8 and 9? 4 and 9 and 9?

35. How many are 5 and 5 and 5? 5 and 5 and 6?
36. How many are 5 and 6 and 6? 5 and 6 and 7?
37. How many are 5 and 7 and 7? 5 and 6 and 8?
38. How many are 5 and 7 and 8? 5 and 8 and 8?
39. How many are 5 and 6 and 9? 5 and 7 and 9?
40. How many are 5 and 8 and 9? 5 and 9 and 9?
41. How many are 6 and 6 and 6? 6 and 7 and 8?
42. How many are 7 and 7 and 7? 7 and 8 and 9?
43. How many are 8 and 8 and 8? 9 and 9 and 9?
44. How many are 1 and 2 and 3 and 4?
45. How many are 2 and 3 and 4 and 5?
46. How many are 1 and 4 and 5 and 6?
47. How many are 2 and 2 and 2 and 2?
48. How many are 2 and 2 and 3 and 3?
49. How many are 2 and 3 and 2 and 3?
50. How many are 3 and 3 and 3 and 3?
51. How many are 3 and 4 and 3 and 4?
52. How many are 3 and 4 and 4 and 5?
53. How many are 3 and 4 and 5 and 6?
54. How many are 4 and 5 and 6 and 7?
55. How many are 1 and 3 and 4 and 5 and 6?
56. How many are 2 and 4 and 6 and 8 and 10?
57. How many are 3 and 5 and 7 and 9 and 11?
58. How many are 2 and 5 and 8 and 4 and 11?
59. How many are 2 and 2 and 3 and 3 and 4?
60. How many are 3 and 4 and 5 and 6 and 7?
61. How many are 4 and 1 and 3 and 5 and 6?
62. How many are 5 and 4 and 6 and 3 and 7?
63. How many are 6 and 1 and 7 and 2 and 3?
64. How many are 7 and 2 and 8 and 3 and 4?
65. How many are 8 and 3 and 9 and 2 and 1?
66. How many are 9 and 4 and 8 and 5 and 4?
67. How many are 10 and 1 and 9 and 2 and 3?
68. How many are 10 and 3 and 7 and 5 and 5?
69. How many are 10 and 9 and 8 and 7 and 6?
70. How many are 9 and 8 and 7 and 6 and 5?
71. How many are 8 and 7 and 6 and 5 and 4?
72. How many are 7 and 6 and 5 and 4 and 3?

CHAPTER III.

EXERCISES IN SUBTRACTION.

LESSON I.

1. If you had 3 cents and spent 1 of them, how many would you then have?

ANS. As many as, with the 1 cent spent, would equal the 3 cents which I had at first. Now, since 1 and 2 are 3, 1 from 3 leaves 2; therefore I should have 2 cents left.

2. What is the act of taking from a number called?

ANS. Taking from a number is called subtraction.

3. What is that called which remains?

ANS. That which remains is called the *remainder*.

4. How many are 3 less 1?

MODEL OF ANSWER.—3 less 1 are 2.

5. How many are 4 less 1? 5 less 1? 6 less 1? 7 less 1? 8 less 1? 9 less 1? 10 less 1?

6. How many are 2 less 2? 3 less 2? 4 less 2? 5 less 2? 6 less 2? 7 less 2? 8 less 2? 9 less 2? 10 less 2? 11 less 2?

7. If you had only 3 cents in your pocket, and took out 3 cents, how many would be left in the pocket?

SOLUTION.—Since 3 from 3 leaves 0, none would be left.

8. If you had 4 cents in your pocket, and took out 3 cents, how many would be left in the pocket?

9. Martha bought 6 sheets of paper; after she had used 3 sheets, how many had she left?

10. Merwin had 8 marbles and lost 3 of them; how many marbles had he left?

11. There were 10 houses in a village, but 3 of them were burned; how many remained?

12. I bought 12 eggs, but broke 3 of them going home; how many remained whole?

13. I had 5 dollars, but paid out 3 dollars for a hat; how much money had I then?

14. Henry had 7 weeks' vacation; after 3 weeks had gone, how many were to come?

15. A man worth 9 thousand dollars, lost 3 thousand; how many thousand had he left?

16. Mr. Hall bought a house for 11 hundred dollars, and sold it for 3 hundred dollars less; how much did he get for the house?

17. A family used 4 gallons of syrup out of a full 5 gallon keg; how many gallons remained?

18. Of 7 barrels of apples, the family used 3 barrels and threw away 1 barrel of rotten ones; how many barrels of apples had they left?

19. Of 12 bushels of potatoes in a bin, 4 bushels were used; how many bushels were left?

20. If of 6 men 4 leave, how many remain?

21. If of 8 birds 4 fly away, how many remain?

22. If of 10 chickens 4 die, how many survive?

23. If of 9 trees 4 fall, how many remain standing?

24. If of 11 words you miss 4, how many do you spell?

25. If of 13 cents you lose 4, how many have you?

26. If of 11 pupils 5 are boys, how many are girls?

27. If 9 boys play ball, and 5 are on one side, how many are on the other?

28. If of 6 acres I sell 5, how many do I keep?

29. If of 7 children 5 die, how many survive?

30. If of 10 cows I sell 5, how many do I keep?

31. If of 8 dollars I spend 5, how many have I?

32. If of 12 days 5 are gone, how many are to come?

33. Peter started to walk 14 miles, but, after walking 5, rode the rest of the way; how far did he ride?

34. James owed for 13 weeks' board, and paid for 5 weeks; for how many did he still owe?

LESSON II.

1. How much difference is there between 3 cents and 5 cents?

ANS. As much as, added to 3 cents, will make 5 cents; that is, 2 cents.

2. How is the difference between two numbers found?

ANS. By taking from the greater a quantity equal to the less.

3. How many more are 7 pupils than 6 pupils?

SOLUTION.—Since 6 pupils taken from 7 pupils would leave 1 pupil, 7 pupils are 1 more than 6 pupils.

4. Ida is 8 years old, and her sister 6; what is the difference between their ages?

5. If Margaret has 10 cents and Sarah 6, how much more money has Margaret than Sarah?

6. If you put a 6-inch rule evenly on a 12-inch rule, how much will the latter project beyond it?

7. If one man is worth 9 thousand dollars, and another 6 thousand, how much has one more than the other?

8. If a coat costs 14 dollars and a vest 6 dollars, what is the difference in their cost?

9. If one kind of ribbon is 15 cents a yard, and another 8, how much cheaper is one than the other?

10. How much cheaper are eggs at 6 cents a dozen than at 13 cents a dozen?

11. How much less price has a melon at 6 cents than one at 11 cents?

12. What is the difference in cost between a ball at 7 cents and one at 10 cents?

13. What is the difference in price between a colt at 9 dollars and one at 7 dollars?

14. If one has a salary of 8 hundred dollars, and another 7 hundred, what is the difference of their salaries?

15. If William earns 12 dollars a month and Jacob 7, what is the difference between their wages?

16. Beef was 11 cents a pound in April, and 7 cents in May; how much did its price fall?

17. If boarding costs one man 14 dollars a week, and another 7, how much does one pay more than the other?

18. One candidate received 13 hundred votes, and another 7 hundred; what was the majority?

19. If one church costs 15 thousand dollars, and another 7 thousand, what is the difference of cost?

20. One year a man got 16 bushels of apples from his orchard, and the next year 7 bushels; what was the difference in the crop of the two years?

21. If one country has 8 millions of people, and another 10 millions, what is the difference between them as to population?

22. If the government of one country costs 12 millions of dollars a year, and that of another 8 millions, how much does one cost more than the other?

23. One army of 8 thousand men met another of 9 thousand; what was the difference of the forces?

24. One man in a speculation made 11 thousand dollars, and another 8 thousand; what was the difference in their gains?

25. One man lost in business 14 thousand dollars, and another 8 thousand; what was the difference in their losses?

26. A man began business with 8 hundred dollars, and closed with 15 hundred; how much did he make?

27. If a boy's age is 13 years, what was his age 8 years ago?

28. In how many years will a girl 8 years old be 16 years old, if she lives?

29. In how many years will she be 17 years old?

30. Sold wood to a merchant for 12 dollars, and took 9 dollars' worth of his goods; how much did the merchant still owe me?

31. Bought 15 dollars' worth of meat of a butcher, and paid him 9 dollars' worth of flour; how much did I still owe him?

32. How many are 8 and 5, less 9?

33. How many are 6 and 7, less 9?

LESSON III.

1. What word is sometimes used instead of the word *less*, to express subtraction?

ANS. *Minus*, a Latin word signifying *less*.

2. If I *write* the question, "How many are $4 - 2$?" what is meant by that sign between 4 and 2?

ANS. It is the sign for *minus*, and the question is read, "How many are 4 minus 2?"

3. How many are $16 - 9$?

MODEL OF ANSWER.—16 minus 9 are 7.

4. How many are $11 - 9$? $14 - 9$? $12 - 9$?

5. How many are $18 - 9$? $17 - 9$? $10 - 9$?

6. How many are $15 - 10$? $12 - 10$? $18 - 10$?

7. How many are $14 - 10$? $11 - 10$? $17 - 10$?

8. How many are $13 - 10$? $10 - 10$? $16 - 10$?

9. How many are $19 - 10$? $20 - 10$? $21 - 10$?

10. Take 2 from 2, 12, 22, 32, 42, 52, 62, 72, 82, 92.

MODEL OF ANSWER.—2 from 2 leaves 0; 2 from 12 leaves 10; 2 from 22 leaves 20; 2 from 32 leaves 30; 2 from 42 leaves 40; 2 from 52 leaves 50; 2 from 62 leaves 60; 2 from 72 leaves 70; 2 from 82 leaves 80; 2 from 92 leaves 90.

11. Take 2 from 3, 13, 23, 33, 43, 53, 63, 73, 83, 93.

12. Take 2 from 4, 14, 24, 34, 44, 54, 64, 74, 84, 94.

13. Take 2 from 5, 15, 25, 35, 45, 55, 65, 75, 85, 95.

14. Take 2 from 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.

15. Take 2 from 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.

16. Take 2 from 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.

17. Take 2 from 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.

18. Take 2 from 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.

19. Take 2 from 11, 21, 31, 41, 51, 61, 71, 81, 91, 101.

20. Take 3 from 3, 13, 23, 33, 43, 53, 63, 73, 83, 93.

21. Take 3 from 4, 14, 24, 34, 44, 54, 64, 74, 84, 94.

22. Take 3 from 5, 15, 25, 35, 45, 55, 65, 75, 85, 95.

23. Take 3 from 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.

24. Take 3 from 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.

25. Take 3 from 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.

26. Take 3 from 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.

27. Take 3 from 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
28. Take 3 from 11, 21, 31, 41, 51, 61, 71, 81, 91, 101.
29. Take 3 from 12, 22, 32, 42, 52, 62, 72, 82, 92, 102.
30. Take 4 from 4, 14, 24, 34, 44, 54, 64, 74, 84, 94.
31. Take 4 from 5, 15, 25, 35, 45, 55, 65, 75, 85, 95.
32. Take 4 from 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.
33. Take 4 from 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.
34. Take 4 from 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
35. Take 4 from 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
36. Take 4 from 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
37. Take 4 from 11, 21, 31, 41, 51, 61, 71, 81, 91, 101.
38. Take 4 from 12, 22, 32, 42, 52, 62, 72, 82, 92, 102.
39. Take 4 from 13, 23, 33, 43, 53, 63, 73, 83, 93, 103.
40. Take 5 from 5, 15, 25, 35, 45, 55, 65, 75, 85, 95.
41. Take 5 from 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.
42. Take 5 from 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.
43. Take 5 from 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
44. Take 5 from 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
45. Take 5 from 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
46. Take 5 from 11, 21, 31, 41, 51, 61, 71, 81, 91, 101.
47. Take 5 from 12, 22, 32, 42, 52, 62, 72, 82, 92, 102.
48. Take 5 from 13, 23, 33, 43, 53, 63, 73, 83, 93, 103.
49. Take 5 from 14, 24, 34, 44, 54, 64, 74, 84, 94, 104.
50. Take 6 from 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.
51. Take 6 from 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.
52. Take 6 from 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
53. Take 6 from 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
54. Take 6 from 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
55. Take 6 from 11, 21, 31, 41, 51, 61, 71, 81, 91, 101.
56. Take 6 from 12, 22, 32, 42, 52, 62, 72, 82, 92, 102.
57. Take 6 from 13, 23, 33, 43, 53, 63, 73, 83, 93, 103.
58. Take 6 from 14, 24, 34, 44, 54, 64, 74, 84, 94, 104.
59. Take 6 from 15, 25, 35, 45, 55, 65, 75, 85, 95, 105.
60. Take 7 from 7, 17, 27, 37, 47, 57, 67, 77, 87, 97.
61. Take 7 from 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
62. Take 7 from 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
63. Take 7 from 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
64. Take 7 from 11, 21, 31, 41, 51, 61, 71, 81, 91, 101.

LESSON IV.

1. Take 7 from 12, 22, 32, 42, 52, 62, 72, 82, 92, 102.
2. Take 7 from 13, 23, 33, 43, 53, 63, 73, 83, 93, 103.
3. Take 7 from 14, 24, 34, 44, 54, 64, 74, 84, 94, 104.
4. Take 7 from 15, 25, 35, 45, 55, 65, 75, 85, 95, 105.
5. Take 7 from 16, 26, 36, 46, 56, 66, 76, 86, 96, 106.
6. Take 8 from 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.
7. Take 8 from 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
8. Take 8 from 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
9. Take 8 from 11, 21, 31, 41, 51, 61, 71, 81, 91, 101.
10. Take 8 from 12, 22, 32, 42, 52, 62, 72, 82, 92, 102.
11. Take 8 from 13, 23, 33, 43, 53, 63, 73, 83, 93, 103.
12. Take 8 from 14, 24, 34, 44, 54, 64, 74, 84, 94, 104.
13. Take 8 from 15, 25, 35, 45, 55, 65, 75, 85, 95, 105.
14. Take 8 from 16, 26, 36, 46, 56, 66, 76, 86, 96, 106.
15. Take 8 from 17, 27, 37, 47, 57, 67, 77, 87, 97, 107.
16. Take 9 from 9, 19, 29, 39, 49, 59, 69, 79, 89, 99.
17. Take 9 from 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
18. Take 9 from 11, 21, 31, 41, 51, 61, 71, 81, 91, 101.
19. Take 9 from 12, 22, 32, 42, 52, 62, 72, 82, 92, 102.
20. Take 9 from 13, 23, 33, 43, 53, 63, 73, 83, 93, 103.
21. Take 9 from 14, 24, 34, 44, 54, 64, 74, 84, 94, 104.
22. Take 9 from 15, 25, 35, 45, 55, 65, 75, 85, 95, 105.
23. Take 9 from 16, 26, 36, 46, 56, 66, 76, 86, 96, 106.
24. Take 9 from 17, 27, 37, 47, 57, 67, 77, 87, 97, 107.
25. Take 9 from 18, 28, 38, 48, 58, 68, 78, 88, 98, 108.
26. Take 10 from 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
27. Take 10 from 11, 21, 31, 41, 51, 61, 71, 81, 91, 101.
28. Take 10 from 12, 22, 32, 42, 52, 62, 72, 82, 92, 102.
29. Take 10 from 13, 23, 33, 43, 53, 63, 73, 83, 93, 103.
30. Take 10 from 14, 24, 34, 44, 54, 64, 74, 84, 94, 104.
31. Take 10 from 15, 25, 35, 45, 55, 65, 75, 85, 95, 105.
32. Take 10 from 16, 26, 36, 46, 56, 66, 76, 86, 96, 106.
33. Take 10 from 17, 27, 37, 47, 57, 67, 77, 87, 97, 107.
34. Take 10 from 18, 28, 38, 48, 58, 68, 78, 88, 98, 108.
35. Take 10 from 19, 29, 39, 49, 59, 69, 79, 89, 99, 109.

36. If you buy a pen for 5 cents and a pencil for 5 cents, and offer a 25 cent piece, what change is due?

37. If you buy a buggy for 10 dollars, pay 10 dollars for repairs, and sell it for 25 dollars, what do you gain?

38. A man sold a horse for 75 dollars, which was 10 dollars more than he cost; how much did he cost?

39. Gave 4 dollars for one barrel of flour and 5 dollars for another; sold both for 12 dollars; what did I gain?

40. Bought a watch for 33 dollars, gave 5 dollars for repairs, and sold it for 10 dollars; what did I lose?

41. John had 10 cents, then received 10 more, then paid out 9 cents; how many had he left?

42. There were 40 sheep in one field, and 30 in another; then 10 jumped from the former into the latter; how many were then in each?

43. Mr. White received 5 hundred votes, and Mr. Black 3 hundred; what was White's majority? How many must change from White to Black to give Black the same majority in the same total number of votes?

44. Samuel has 8 cents and Joseph 4 cents; how many must Samuel give Joseph to cause Joseph to have as many more than Samuel as Samuel now has more than he?

45. An army of 15 thousand men met another of 9 thousand; the former lost 3 thousand and the latter 1 thousand in the battle; how much less excess has the former now over the latter, than before the battle? If 2 thousand men now desert from the former to the latter, how does it leave the armies?

46. A man having 67 dollars, made 9 more, then lost 6 dollars; how many had he then?

47. A merchant began business with 12 hundred dollars, and the first year lost 5 hundred, but the second year made 9 hundred; what was he then worth?

48. From a piece of cloth containing 44 yards, were sold 9 yards, then 8, then 7, then 6; how many were left?

49. My property is worth 58 thousand dollars, but I owe Mr. Jones 10 thousand, Mr. Green 5 thousand, and Mr. Smith 7 thousand; what am I worth?

CHAPTER IV.

EXERCISES IN MULTIPLICATION.

LESSON I.

1. How many are 2 and 2 and 2 and 2 and 2?

ANS. 2 and 2 are 4, and 2 more are 6, and 2 more are 8, and 2 more are 10; therefore the sum of five 2's is 10.

2. What is mentioning a number two or more times called?

ANS. It is called *repeating* it.

3. How can the sum of five 2's be more briefly stated than by adding?

ANS. By saying, "5 times 2 are 10."

4. What is this short way of stating; and finding the sum of the repetitions of a number called?

ANS. It is called *Multiplication*.

5. How many are two 5's?

MODEL OF ANSWER.—Two 5's are 10.

REMARK.—The learner should actually work out by Addition the proof of every answer in Multiplication. He may assist himself by making groups of marks and counting them. Thus, for the above question, let him make five marks twice, and count them, to get the sum of two 5's. His already acquired knowledge of addition will save this trouble in most cases. In *recitation*, he need only state his results.

6. How many are four 2's? Two 4's? Three 2's? Two 3's? Two 2's? Six 2's? Two 6's? Seven 2's? Two 7's? Eight 2's? Two 8's? Nine 2's? Two 9's? Ten 2's? Two 10's? Eleven 2's? Two 11's? Twelve 2's?

7. How many are 3 times 3?

MODEL OF ANSWER.—3 times 3 are 9.

8. How many are 4 times 3? 3 times 4? 5 times 3? 3 times 5? 6 times 3? 3 times 6? 7 times 3? 3 times 7? 8 times 3? 3 times 8? 9 times 3? 3 times 9? 10 times 3? 3 times 10? 11 times 3? 3 times 11? 12 times 3? 3 times 12? 3 times 1?

9. If I *write* the question, " 4×2 ?" what is meant by that sign between 4 and 2?

ANS. It is the sign of *multiplication*, and the question may be read, "How many are 4 times 2?"

REMARK.—Some read such expressions thus:—"4 *multiplied into* 2," or briefly, "4 *into* 2."

10. How many are 4×4 ? 5×4 ? 4×5 ? 6×4 ? 4×6 ? 7×4 ? 4×7 ? 8×4 ? 4×8 ? 9×4 ? 4×9 ? 10×4 ? 4×10 ? 11×4 ? 4×11 ? 12×4 ? 4×12 ? 4×1 ?

11. What cost 5 books at 1 dollar apiece?

ANALYSIS.—If 1 book costs 1 dollar, 5 such books will cost 5 times 1 dollar, or 5 dollars.

12. What cost 5 hats at 5 dollars apiece?

13. What cost 6 melons at 5 cents apiece? Five melons at 6 cents apiece?

14. What cost 7 bottles of ink at 5 cents each? Five pencils at 7 cents each?

15. What cost 9 sheep at 5 dollars apiece? Five heifers at 9 dollars apiece?

16. How many hills are in a potato-patch of 5 rows, 8 hills in a row? Reckon them the other way.

17. How many men in a company of 10 platoons, 5 in a platoon? Of 5 files, 10 in each file?

18. How many trees in an orchard of 12 rows, 5 in each row? Reckon them the other way.

19. What cost 5 pounds of sugar at 11 cents a pound? Eleven pounds of rice at 5 cents a pound?

20. Julius worked 6 days at 6 cents a day; what wages did he receive for the week's work?

21. At 6 cents a quart what cost 7 quarts of syrup? Six quarts of cider at 7 cents a quart?

LESSON II.

1. At 6 miles an hour how far does a coach go in 8 hours? How far in 6 hours, at 8 miles an hour?

2. At 6 dollars a thousand what cost 9 thousand bricks? Six thousand at 9 dollars a thousand?

3. If a barrel of flour lasts 10 persons 6 weeks, how long will it last 1 person?

ANALYSIS.—Since it would last 1 person 10 times as long as it would last 10 persons, it would last 1 person 10 times 6 weeks, or 60 weeks.

4. If 6 men dig a ditch in 10 days, how long would it take 1 man to dig it?

5. If 11 men earn a hundred dollars in 6 days, how long would it take 1 of them to earn it?

6. If a bin of oats lasts 6 horses 11 weeks, how long would it last 1 horse?

7. At 6 dollars a week what costs 12 weeks' board? Six weeks' board at 12 dollars a week?

8. If a certain heap of coal supplies 7 fires 7 weeks, how long would it have supplied 1 fire?

9. Two men start from the same place and travel in opposite directions, one 4 miles an hour, the other 3; how far apart will they be at the end of 8 hours? 9 hours? 10 hours? 11 hours? 12 hours?

10. If two persons start together and travel in the same direction, one 4 miles an hour, the other 12, how far apart will they be in 8 hours? 10 hours? 11 hours? 12 hours? 9 hours?

11. What cost 7 yards of cloth at 8 cents a yard?

12. What cost 7 quires of paper at 10 cents a quire?

13. If a barrel of flour lasts 9 persons 7 weeks, how many persons will it last 1 week?

ANALYSIS.—Since, to consume the flour in 1 week, it will take 7 times as many persons as will consume it in 7 weeks, it will last 7 times 9 persons, which are 63 persons, 1 week.

14. If a stack of hay lasts 12 cows 7 weeks, how many cows would it last 1 week?

15. If a cask of water lasts 11 sailors 7 days, how many would it last 1 day?

16. What cost 8 yards of ribbon at 9 cents a yard?

17. If you spend 10 cents a day, to how much will it amount in 8 days?

18. At 11 dollars a thousand, what cost 8 thousand shingles?

19. If 12 men dig a cellar in 8 days, how many men would dig it in 1 day?

20. If the provisions in a fort would last 9 thousand men 9 months, how many men would they support 1 month?

21. How many months would the same provisions support 1 thousand men?

22. At 10 cents a dozen what cost 9 dozen of eggs? What cost 10 dozen at 9 cents a dozen?

23. How many bricks in a pavement having 11 bricks in its length, and 9 in its breadth? Reckon them both ways.

24. Martin sold 9 melons at 12 cents apiece; what did he get for all?

25. He sold 12 quarts of grapes at 9 cents a quart; what did he get for all?

26. If 10 cents make a dime, how many cents are there in 10 dimes?

27. If 10 dimes make a dollar, how many dimes are there in 11 dollars?

28. If 12 boys give 10 cents apiece for the use of a boat, what is the amount paid by all?

29. If each of 10 persons pays 11 dollars school-tax, what is the amount of all the taxes?

30. How many pins in 10 rows, 12 in each row?

31. If you spell 11 words a day for 12 days, how many do you spell in all?

32. If you write 11 lines a day for 11 days, how many lines in all do you write?

33. What cost 11 pounds of coffee at 12 cents a pound?

34. What cost 12 pounds of butter at 12 cents a pound?

CHAPTER V.

EXERCISES IN DIVISION.

LESSON I.

1. If there were six eggs in a basket, how many times could you take out two eggs?

ANS. Three times, because taking the first two would leave *four*, taking the second two would leave *two*, and taking the third two would leave nothing.

2. That is reasoning by subtraction; can you show the same by addition or multiplication?

ANS. The first time *two* would be taken, the second time two more, making *four*, the third time two more, making *six*, or all. Or, since three 2's are 6, 2 is contained in 6 three times.

3. In these operations what was done with *six*?

ANS. Six was *divided* into parcels, of *two* each.

4. What, then, is Division?

ANS. Division is the process by which we find how many times one number contains another.

5. How many times does 6 contain 2?

MODEL OF ANSWER.—Since 3 times 2 are 6, 6 contains 2 three times.

6. How many times does 4 contain 2?

7. How many times does 10 contain 2?

8. How many times does 8 contain 2?

9. How many times does 14 contain 2?

10. How many times does 12 contain 2?

11. How many times does 18 contain 2?

12. How many times does 16 contain 2?

• 13. How many times does 20 contain 2?

14. How many times does 6 contain 3?

ANS. If 6 is divided into parts of 3 each, there will be two such parts; therefore 6 contains 3 two times.

15. When two equal parts make a quantity, what is each part called?

ANS. Each of two equal parts which make a quantity is called *one-half* of that quantity; therefore, since two 3's are 6, 3 is one-half of 6.

16. How many times does 9 contain 3?

17. When three equal parts make a quantity, what is each part called?

ANS. *One-third* of that quantity; therefore, since three 3's are 9, 3 is one-third of 9.

18. How many times does 12 contain 3?

19. When four equal parts make a quantity, what is each part called?

ANS. *One-fourth* of that quantity; therefore, since four 3's are 12, 3 is one-fourth of 12.

REMARK.—Fourths are sometimes called *quarters*.

20. What part of 15 is 3?

MODEL OF ANSWER.—Because 3 is contained in 15 five times 3 is one-fifth of 15.

21. What part of 18 is 3? Of 24? Of 30? Of 21? Of 33? Of 27? Of 36?

22. At 4 cents apiece, how many pencils can you buy for 8 cents?

ANALYSIS.—If 4 cents buy 1 pencil, 8 cents will buy as many pencils as 4 is contained times in 8. Since 4 is contained 2 times in 8, 8 cents will buy 2 pencils at 4 cents apiece.

23. At 4 dollars apiece, how many hats will 12 dollars buy?

24. At 4 dollars a barrel, how many barrels of flour will 20 dollars buy? How many will 16 dollars buy?

25. If 4 quarts make a gallon, how many gallons are there in 32 quarts?

26. If 4 pecks make a bushel, how many bushels are there in 24 pecks?

27. How many rows, of 4 each, will 28 trees make?

LESSON II.

1. How many sheep are one-half of 24 sheep?

MODEL OF ANSWER.—One-half of 24 sheep is that number which is contained 2 times in 24. Since 12 is contained 2 times in 24, 12 sheep are one-half of 24 sheep.

2. How many sheep are one-half of 22 sheep? Of 18?
20? 12? 8? 14? 10? 16? 6? 4?

3. How many are one-third of 3? 12? 18? 6? 24?
9? 27? 15? 21? 36? 30?

4. How many are one-fourth of 4? 12? 20? 28? 36?
44? 8? 16? 24? 32? 40? 48?

5. How many are one-fifth of 5? 20? 40? 50? 60?
10? 25? 15? 35? 45? 55? 30?

6. How many are one-sixth of 6? 18? 36? 54? 72?
12? 24? 48? 60? 30? 42? 66?

7. How many are one-seventh of 7? 21? 35? 49? 63?
84? 14? 28? 42? 56? 70? 77?

8. How many are one-eighth of 8? 40? 64? 80? 96?
16? 32? 48? 72? 24? 56? 88?

9. How many are one-ninth of 9? 27? 45? 63? 81?
99? 18? 36? 54? 72? 90? 108?

10. How many are one-tenth of 10? 30? 60? 110?
20? 50? 70? 90? 120? 40? 80? 100?

11. How many are one-eleventh of 11? 33? 55? 77?
99? 121? 22? 44? 66? 88? 110? 132?

12. How many are one-twelfth of 12? 24? 36? 48?
60? 72? 84? 96? 108? 120? 132? 144?

13. If 5 pencils cost 20 cents, what costs 1 pencil?

FIRST ANALYSIS.—If 5 pencils cost 20 cents, 1 pencil costs that part of 20 cents which 1 pencil is of 5 pencils. Since 1 pencil is one-fifth of 5 pencils, 1 pencil costs one-fifth of 20 cents, which is 4 cents.

SECOND ANALYSIS.—If 5 pencils cost 20 cents, 1 pencil costs as many cents as 5 is contained times in 20. Since 5 is contained 4 times in 20, 1 pencil costs 4 cents.

NOTE.—When the learner has become familiar with the reasoning in the First Analysis, he can abbreviate as follows:—

If 5 pencils cost 20 cents, 1 pencil costs one-fifth of 20 cents, which is 4 cents.

14. If 5 caps cost 10 dollars, what costs 1 cap?

15. If 5 hats cost 15 dollars, what costs 1 hat?

16. If 5 men, with equal wages, earn 25 dollars a day, what are the daily wages of each?

17. Gave 35 cents for 5 yards of calico; what was the price per yard?

18. Gave 30 dollars for 5 barrels of flour; what was the price per barrel?

19. Paid 45 dollars for 5 weeks' board; what was the price per week?

20. Paid 40 cents for 5 pounds of sugar; what was the price per pound?

21. Paid 50 cents for 5 dozen of eggs; what was the price per dozen?

22. Gave 60 cents for 5 pounds of beef; what was the price per pound?

23. Gave 55 dollars for 5 thousand feet of lumber; what was the price per thousand?

24. Gave 12 dollars for 6 sheep; what was the cost per head?

25. Gave 6 dollars for 6 lambs; what was the price per head?

26. Gave 72 dollars for 6 head of cattle; what was the price per head?

27. Gave 60 dollars for 6 acres of land; what was the cost per acre?

28. Gave 48 cents for 6 pairs of ducks; what was the price per pair?

29. Gave 66 cents for 6 brace of pigeons; what was the price per brace?

30. Paid 18 dollars for 6 reams of paper; what was the cost per ream?

31. Gave 30 cents for 6 pecks of oats; what was the cost per peck?

32. Sold 6 bushels of pears for 24 dollars; what did I get per bushel?

LESSON III.

1. If an orchard of 42 trees has 6 equal rows, how many trees are in each of these rows?

2. If a field of corn, containing 54 hills, has 6 equal rows, how many hills are in each row?

3. If 6 apples are given for an orange, how many oranges can 36 apples buy?

4. If 7 marbles cost as much as a top, how many tops can be bought with the price of 14 marbles? 21 marbles? 28 marbles? 35 marbles?

5. If a man has 56 dollars, and spends 7 dollars a week, how long will the money last?

6. If 7 calves cost 42 dollars, what is that apiece?

7. A number that I think of, multiplied by 7 makes 63; of what number do I think?

8. At 7 cents per yard, how many yards of muslin can be bought for 49 cents? For 70 cents? For 77 cents? For 84 cents?

9. Eight quarts make a peck; how many pecks are there in 16 quarts? 24 quarts? 48? 64? 32? 72? 40? 96? 88? 80? 56?

10. At 9 miles an hour, how long would you be in, going 18 miles? 36 miles? 27 miles?

11. If you travel steadily 45 miles in 9 hours, what is the rate per hour?

12. If your wages are 54 dollars in 9 weeks, what are they per week?

13. If a cow gives 63 quarts of milk in 9 days, what is the rate per day?

14. If a horse eats 72 quarts of oats in 9 days, what is the rate per day?

15. If 9 equal windows have 81 panes of glass, how many panes has each window?

16. At 9 cents a yard, how many yards of ribbon can be bought for 90 cents? 108 cents? 99 cents?

17. If 9 dollars buy 9 hoes, what is the price?

18. If 1 team can plow a piece of land in 20 days, how long would it take 10 equal teams to plow it?

NOTE.—More teams require less time.

19. If, at 1 mile an hour, a boat goes a certain distance in 30 hours, how long would it be in going the same distance at 10 miles an hour?

20. If a crib of corn would last 1 pig 40 days, how long would it last 10 pigs?

21. If, at 1 page a day, a person reads a book in 50 weeks, how long would it take at 10 pages a day?

22. If it takes 1 man 60 days to lay the brick of a house, how long would it take 10 men? 12 men?

23. If 1 pipe fills a cistern in 70 minutes, how long would 10 such pipes be in filling it?

24. If a stack of hay would last 1 cow 80 days, how long would it last 10 cows?

25. If 1 man can mow a field in 90 hours, how long would it take 10 men to mow it?

26. If 1 man can clear a piece of woods in 100 days, how many days would it take 10 men to do it?

27. If 1 man can hoe a field in 110 hours, how long would it take 10 men? 11 men?

28. If 11 boys share equally 11 apples, what part of the whole, and how many, has each boy? If they share 22 apples? 33? 44? 55? 66?

29. If 11 men make up equally a purse of 77 dollars, what part, and how much, does each give? If they give 88 dollars? 110? 99? 121?

30. Hubert is 12 years old, and his infant brother one-twelfth as old; how old is Hubert's brother?

31. At 12 cents apiece, how many meals can a person pay for with 24 cents? With 36 cents? With 72 cents? With 60 cents? With 48 cents?

32. If a person lived on 12 cents a day, how many days would 84 cents last him? 96 cents?

33. A company of 108 men formed 12 equal platoons; how many men were in a platoon?

34. How many 12's in 120? 144? 132?

CHAPTER VI.

HIGHER EXERCISES IN THE SUBJECTS OF THE PRECEDING CHAPTERS.

LESSON I.

1. What do single figures signify?

ANS. Single figures signify single things, that is, *ones*, or *units*.

2. If a figure is the second toward the left, what does it signify?

ANS. Figures in the second place toward the left signify *tens*; thus, at the left of another figure, 1 is 1 *ten*, 2 is 2 *tens*, or *twenty*, 3 is 3 *tens*, or *thirty*, 4 is 4 *tens*, or *forty*, 5 is 5 *tens*, or *fifty*, 6 is 6 *tens*, or *sixty*, 7 is 7 *tens*, or *seventy*, 8 is 8 *tens*, or *eighty*, 9 is 9 *tens*, or *ninety*.

3. How many are 30 and 20?

ANALYSIS.—30 is 3 *tens*, and 20 is 2 *tens*; 3 *tens* and 2 *tens* are 5 *tens*, or 50; therefore 30 and 20 are 50.

4. How many are 20 and 20? 70 and 20? 40 and 20? 80 and 20? 50 and 20? 90 and 20? 60 and 20? 100 and 20?

5. How many are 30 and 30? 60 and 30? 40 and 30? 70 and 30? 50 and 30? 80 and 30? 90 and 30?

6. How many are 40 and 40? 70 and 40? 50 and 40? 60 and 40? 80 and 40?

7. How many are 50 and 50? 70 and 50? 60 and 50? 60 and 60? 70 and 60? 70 and 70?

8. How many *tens* are 80 and 60? 80 and 70? 90 and 80? 80 and 80? 90 and 70? 90 and 90?

9. How many are 21 and 43?

ANALYSIS.—21 is 2 *tens* and 1; 43 is 4 *tens* and 3; 2 *tens* and 4 *tens* are 6 *tens*, or 60; 1 and 3 are 4; 60 and 4 are 64; therefore 21 and 43 are 64.

OR, 21 is 2 *tens* and 1; 43 and 10 are 53; 10 more are 63, and 1 makes 64. Therefore 43 and 21 are 64.

10. How many are 20 and 32? 22 and 23? 23 and 24? 24 and 25? 23 and 25? 22 and 25? 22 and 27?

11. How many are 21 and 35? 33 and 33? 32 and 27? 34 and 45? 25 and 43? 36 and 32? 37 and 42?

12. How many are 44 and 55? 43 and 52? 46 and 31? 47 and 20? 48 and 30? 49 and 40? 45 and 50?

13. How many are 51 and 42? 52 and 33? 57 and 41? 53 and 24? 54 and 45? 55 and 22? 56 and 33?

14. How many are 100 and 200? 200 and 200? 200 and 300? 200 and 700? 200 and 600? 200 and 400? 200 and 500? 200 and 800?

15. How many are 300 and 300? 300 and 600? 300 and 500? 300 and 400? 300 and 700?

16. How many are 400 and 400? 500 and 400?

17. How many are 109 and 9?

ANALYSIS.—9 and 9 are 18; 18 and 100 are 118; therefore 109 and 9 are 118.

18. How many are 108 and 9? 207 and 9? 306 and 9? 405 and 9? 504 and 9? 603 and 9? 702 and 9?

19. How many are 801 and 9? 902 and 8? 703 and 5? 804 and 3? 603 and 3? 702 and 3? 808 and 8?

20. How many are 909 and 7? 707 and 7? 606 and 5? 505 and 4? 404 and 4? 303 and 6? 202 and 6?

21. How many are 130 and 30? 120 and 20? 110 and 50? 440 and 40? 550 and 40? 660 and 20?

22. How many are 740 and 50? 820 and 40? 360 and 30? 270 and 20? 150 and 20?

23. How many are 222 and 222? 234 and 234? 345 and 534? 333 and 333? 444 and 444? 555 and 444? 666 and 222? 456 and 123? 567 and 231? 678 and 120? 304 and 405? 254 and 143? 426 and 252? 542 and 242? 731 and 231? 825 and 124? 633 and 366? 543 and 456?

LESSON II.

1. What is the sum of $25 + 27 + 38$?

NOTE.—If such expressions are written on the blackboard to assist the memory, no other assistance will be needed in the use of the Analysis; but, if not thus written, let the teacher wait till the pupil has added two numbers, before pronouncing the next number. The pupil should use that Analysis which is easiest, and least liable to confuse him. Many prefer the Second Analysis in these respects. If the teacher does not pronounce all the numbers at first, something like the First Analysis will be necessary.

FIRST ANALYSIS.—2 *tens* and 5, and 2 *tens*, are 4 *tens* and 5, or 45; 45 and 7 are 52; 5 *tens* and 2, and 3 *tens*, are 8 *tens* and 2, or 82; 82 and 8 are 90. Therefore $25 + 27 + 38$ are 90.

SECOND ANALYSIS.—2 *tens* and 2 *tens* are 4 *tens*, and 3 *tens* are 7 *tens*, or 70; 70 and 5 are 75, and 7 are 82, and 8 are 90. Therefore $25 + 27 + 38$ are 90.

THIRD ANALYSIS.—2 *tens* and 2 *tens* are 4 *tens*, and 3 *tens* are 7 *tens*; 5 and 7 are 12, and 8 are 20, or 2 *tens*; 7 *tens* and 2 *tens* are 9 *tens*, or 90. Therefore $25 + 27 + 38$ are 90.

2. What is the sum of $16 + 23 + 47$?
3. What is the sum of $19 + 38 + 56$?
4. What is the sum of $24 + 35 + 48$?
5. What is the sum of $36 + 47 + 28$?
6. What is the sum of $16 + 49 + 51$?
7. What is the sum of $13 + 17 + 19$?
8. What is the sum of $15 + 18 + 20$?
9. What is the sum of $54 + 25 + 17$?
10. What is the sum of $67 + 28 + 14$?
11. Gave 19 cents for raisins, 24 cents for lemons, and 31 cents for cake; what did all cost?

ANALYSIS.—All cost as many cents as is the sum of 19, 24, and 31. The 1 *ten* of 19, and 2 *tens* of 24, are 3 *tens*, and the 3 more *tens* of 31 are 6 *tens*, or 60; 60, and the 9 of 19, are 69, and the 4 of 24 are 73, and the 1 of 31 are 74. Therefore all cost 74 cents.

NOTE.—Make an analysis from the other models, if preferred.

12. Gave 23 dollars for a carpet, 35 dollars for a secre-

tary, and 28 dollars for a sofa; what was the sum of these outlays?

13. A man, 42 years old, has a wife 39 years old, and a son 18 years old; what is the sum of their ages?

14. Sold corn for 17 dollars, wheat for 26 dollars, and oats for 32 dollars; what did I get for all?

15. Gave 75 dollars for a horse, 25 dollars for a cow, and 18 dollars for hay; how much for all?

16. Drove a horse 42 miles one day, 37 miles the next, and 27 the next; how far did I drive in all?

17. Spent 37 dollars in April, 29 in May, and 48 in June; how much did I spend in the three months?

18. Made 46 dollars in July, 38 in August, and 36 in September; how much did I make in those three months?

19. Raised 52 bushels of wheat, 23 of turnips, and 35 of rye; how many bushels were these crops?

20. Begin at 2, and count 100 by *twos*.

21. Begin at 3, and count 101 by *twos*.

22. Begin at 3, and count 102 by *threes*.

23. Begin at 4, and count 100 by *threes*.

24. Begin at 5, and count 101 by *threes*.

25. Begin at 4, and count 100 by *fours*.

26. Begin at 5, and count 101 by *fours*.

27. Begin at 6, and count 102 by *fours*.

28. Begin at 7, and count 103 by *fours*.

29. Begin at 5, and count 100 by *fives*.

30. Begin at 6, and count 101 by *fives*.

31. Begin at 7, and count 102 by *fives*.

32. Begin at 8, and count 103 by *fives*.

33. Begin at 9, and count 104 by *fives*.

34. Begin at 6, and count 102 by *sixes*.

35. Begin at 7, and count 103 by *sixes*.

36. Begin at 7, and count 105 by *sevens*.

37. Begin at 8, and count 106 by *sevens*.

38. Begin at 9, and count 105 by *eights*.

39. Begin at 11, and count 99 by *eights*.

40. Begin at 10, and count 100 by *nines*.

LESSON III.

1. Conrad had 30 cents, and spent 20 of them; how many cents had he left?

ANALYSIS.—30 is 3 *tens*, and 20 is 2 *tens*; 2 *tens* from 3 *tens* leave 1 *ten*, or 10. Therefore he had 10 cents left.

2. Having 120 dollars, how many would be left if I spent 20? 30? 40? 50? 60? 70? 80? 90?

3. Having 110 sheep, how many would be left if I sold 20? 30? 40? 50? 60? 70? 80? 90?

4. A merchant has 100 barrels of flour; how many would be left after selling 20? 30? 40? 50? 60? 70? 80? 90?

5. B has 90 bushels of potatoes; how many would be left after using 20 bushels? 30? 40? 50? 60? 70? 80?

6. C has 80 bushels of apples; how many would be left after using 20 bushels? 30? 40? 50? 60? 70?

7. From 70 take 20, 30, 40, 50, 60.

8. Of 74 cents I spent 28; how many were left?

ANALYSIS.—Taking 20 from 74 is taking 2 *tens* from 7 *tens* and 4, which leaves 5 *tens* and 4, or 54; taking 8 more than 20 from 74 leaves 8 *less* than 54, which is 46. Therefore 46 cents were left.

9. Of 67 persons 29 died; how many survived?

10. With 34 dollars D traded till he had 83 dollars; how much did he make?

11. If A owes B 48 dollars, and B owes A 92 dollars, what must B pay A at settlement?

12. If you owed 65 dollars, and paid 36 dollars, how much would you still owe?

13. How many are 53 less 18? 57 less 37? 61 less 32? 72 — 43? 84 — 26? 95 — 68?

14. After a man has gone 39 miles on a journey of 97 miles, how far has he yet to go?

15. Bought at 45, sold at 63; what was the gain?

16. Bought at 50, sold at 34; what was the loss?

17. The greater of two numbers is 89, and the less is 45; what is the difference?

18. The greater of two numbers is 73, and the difference is 36; what is the less number?

19. How much must 32 be increased to equal 61?

20. The sum of two numbers is 47, and the less is 19; what is the greater number?

21. The sum of two numbers is 55, and the greater is 28; what is the less number?

22. What is the difference between 75, and the sum of 18 and 38?

ANALYSIS.—The 1 *ten* of 18, and the 3 *tens* of 38, make 4 *tens* or 40; 40, and the 8 of 18, are 48, and the 8 more of 38 are 56; therefore the sum of 18 and 38 is 56. Now, taking the 50 of 56 from 75 is taking 5 *tens* from 7 *tens* and 5, which leaves 2 *tens* and 5, or 25; and taking the 6 more of 56 from 75 leaves 6 less than 25, which is 19; therefore the difference is 19.

23. From 60 take the sum of 22 and 17.

24. From a piece of cloth, containing 44 yards, were cut first 13 yards, then 12 yards; how many yards were left?

25. From a barrel of cider, containing 37 gallons, were drawn 7, 10, and 12 gallons; how much remained?

26. Bought muslin for 32 cents, and gloves for 25 cents, and offered three 25 cent pieces; how much change should I receive?

27. John, having 50 cents, bought a sled for 15 cents, and mittens for 18 cents; how much had he left?

28. A man went from home 33 miles the first day, and 29 miles farther the next day; the third day he returned 35 miles; how far was he yet from home?

29. To raise 100 dollars one man subscribed 10 dollars, another 25, another 30, and the fourth man made up the rest; what did he give?

30. Gave 50 dollars for a horse, and 30 for a harness; sold both for 75 dollars; what was the loss?

31. A's horse cost 90 dollars, and his keeping 30 dollars; his work brought A 100 dollars, and A sold him for 30 dollars; what did A gain?

32. From 18 take 12 less 2.

SOLUTION.—12 less 2 are 10; 10 from 18 leaves 8.

33. From 25 take 19 less 8.

34. From the sum of 20 and 15 take the difference of 27 and 12.

35. From $32 + 28$ take $17 + 18$.

36. E ran in debt to F 30 dollars, and afterward 25 dollars. He paid him at one time 20 dollars, at another 15; how much did E then owe F?

37. From the difference of 75 and 43, take the difference of 98 and 84.

38. A agreed to do a job for 50 dollars, but his employer allowed him 15 dollars more. A paid for work 35 dollars, and for stuff 12 dollars; how much did A make by the job?

39. B agreed to do a job for 75 dollars, but failed to get his pay by 13 dollars. He paid out 27 dollars for work, and 17 for stuff; how much did he make?

40. C agreed to do a job for 100 dollars, but received 18 dollars less. The expenses of the job were 90 dollars; how much did he lose?

41. A merchant bought a bale of linen for 65 dollars, and paid 9 dollars for transportation. Being damaged, he sold it for 70 dollars; what did he lose?

42. Begin at 100 and count downward by 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's and 10's.

43. Begin at 101 and count downward by 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's and 10's.

44. Begin at 102 and count downward by 3's, 4's, 5's, 6's, 7's, 8's, 9's and 10's.

45. Begin at 103 and count downward by 4's, 5's, 6's, 7's, 8's, 9's and 10's.

46. Begin at 104 and count downward by 5's, 6's, 7's, 8's, 9's and 10's.

47. Begin at 105 and count downward by 6's, 7's, 8's, 9's and 10's.

48. Begin at 106 and count downward by 7's, 8's, 9's.

LESSON IV.

1. How many are 13 times 10?

SOLUTION.—12 *tens* are 120, and 13 *tens* are 10 more, or 130

2. How many are 14 *tens*? 15 *tens*? 16 *tens*? 17 *tens*?
18 *tens*? 19 *tens*? 20 *tens*? 21 *tens*? 22 *tens*? 23 *tens*?
24 *tens*? 25 *tens*? 26 *tens*? 27 *tens*? 28 *tens*? 29 *tens*?
30 *tens*? 31 *tens*?

3. What figure, annexed to a number, makes it express as many *tens* as, before, it expressed *units*?

4. How many are 5 times 17?

SOLUTION.—17 is 1 *ten* 7 *units*; 5 times 7 are 35, which is 3 *tens* 5 *units*; 5 times 1 *ten* are 5 *tens*; 5 *tens* and 3 *tens* and 5 are 8 *tens* and 5, or 85; therefore 5 times 17 are 85.

5. How many are 2×16 ? 2×17 ? 2×18 ? 2×19 ?
 2×26 ? 2×37 ? 2×48 ? 2×59 ?

6. How many are 9×15 ? 7×26 ? 5×37 ? 4×48 ?
 3×59 ? 2×77 ? 3×88 ? 6×56 ?

7. How many are 9×16 ? 8×17 ? 7×18 ? 6×27 ?
 5×36 ? 4×45 ? 3×54 ? 2×62 ?

8. What cost 8 cows at 18 dollars a head?

ANALYSIS.—If 1 cow costs 18 dollars, 8 cows, at the same price, cost 8 times 18 dollars. 18 is 1 *ten* 8 *units*. 8 times 8 are 64, which is 6 *tens* 4 *units*. 8 times 1 *ten* are 8 *tens*. 8 *tens* and 6 *tens* and 4 are 14 *tens* and 4, or 144. Therefore 8 cows, at 18 dollars a head, cost 144 dollars.

9. What cost 6 horses, at 76 dollars apiece?

10. What cost 9 tons of hay, at 25 dollars a ton?

11. What cost 8 tons of iron, at 33 dollars a ton?

12. What is 12 months' pay, at 28 dollars a month?

13. At 37 miles an hour, how far does a train move in 11 hours?

14. At 54 dollars an acre, what cost 10 acres?

15. What cost 7 books at 13 cents apiece?

16. What cost 5 slates at 19 cents apiece?

17. How much oil is in 4 casks of 56 gallons each?

18. What cost 3 knives at 25 cents apiece?

LESSON V.

1. If a stack of hay would last 15 horses 12 days, how long would it last 1 horse?

2. If a bin of oats would last 24 horses 11 days, how long would it last 1 horse?

3. If a stock of provisions is sufficient for 35 men 10 days, how long would it last 1 man?

4. If it would take 46 men 9 days to cut down a certain piece of woods, how long would it take 1 man?

5. If it would take 57 men 8 hours to hoe a certain field, how long would it take 1 man?

6. If a certain sum would pay the wages of 7 men for 68 days, how many men would it pay for 1 day?

7. For how many days would it pay 1 man's wages?

8. What cost 6 chairs at 4 dollars apiece, and 2 tables at 8 dollars apiece?

9. What cost 5 pounds of coffee at 15 cents a pound, and 3 pounds of sugar at 8 cents a pound?

10. What cost 4 sofas at 25 dollars apiece, and 7 beds at 14 dollars apiece?

11. If two trains leave a place at the same time in opposite directions, one going 18 miles an hour, the other 22 miles an hour, how far apart will they be in 2 hours? In 3 hours? In 4 hours? In 5 hours?

12. John is 12 years old, and his father 3 times as old; what is the sum of their ages?

13. If a man earns 35 dollars a month, and his son 8 dollars a month, what do both earn in 6 months?

14. What is the sum of the products 7×7 , 5×8 , and 8×4 ?

NOTE.—The result of multiplication is called a *product*, and the numbers multiplied to produce it are called *factors*.

15. Bought 6 oranges at 5 cents apiece, 2 pounds of almonds at 25 cents a pound, and 3 pounds of raisins at 18 cents a pound; what did all cost?

16. Bought 9 barrels of flour at 8 dollars a barrel, and sold it for 90 dollars; what did I gain?

ANALYSIS.—The gain was as much as 90 dollars, the selling price, exceeds 9 times 8 dollars, or 72 dollars, the cost. Since 72 from 90 leaves 18, the gain was 18 dollars.

17. Bought 7 pounds of coffee at 12 cents a pound, and offered a dollar (100 cents); how much change should I receive?

18. A farmer sold a merchant 6 pounds of butter at 16 cents a pound, and received 10 pounds of mackerel at 12 cents a pound; who was in debt, and how much?

19. A lady went a-shopping with 65 dollars; and bought 11 yards of silk at 2 dollars a yard, 2 pairs of shoes at 2 dollars a pair, and 7 yards of cloth at 5 dollars a yard; how much money had she left?

20. William is 14 years old, and his father lacks 2 years of being 3 times as old; what is the father's age, and the sum of their ages?

21. If a man's wages are 30 dollars a month, and his expenses are 20 dollars a month, how much will he save in 10 months? In 12 months?

22. James is 11 years old, and his father is 5 years more than three times as old; what is the father's age, and the difference of their ages?

23. A man bought a 40-gallon cask of wine at 2 dollars a gallon; 10 gallons having leaked out, he sold the rest at 3 dollars a gallon. Did he make, or lose, and how much?

24. If 2 pears are worth 4 apples, and 4 apples are worth 8 plums, how many plums will 6×2 pears buy?

25. How many are 9×3 , multiplied by 2×4 ?

26. From 7×9 take 5×11 .

27. How many are 6×8 , plus 7×5 ?

28. Saturday night Mr. Smith pays his "hands" for 6 days' work, viz. 3 men at 10 dimes a day, 5 women at 6 dimes a day, and 6 boys at 4 dimes a day; how many dimes does he pay out for wages per week?

29. How many are $2 \times 3 \times 4 \times 5$?

30. How many are $4 \times 5 \times 6 - 2 \times 5 \times 6$?

LESSON VI.

1. If I *write* the question, "How many are $18 \div 2$?" what is the sign between the 18 and 2?

ANS. It is the sign of division, and signifies that the number before it is to be divided by the number after it. The written question is read, "How many are 18 divided by 2?"

2. What does the expression, "18 *divided by* 2," mean?

ANS. It means either the division of 18 into 2's, that is, into parts of 2 each, to find their *number*, which is 9, or the division of 18 into 2 equal parts, to find their *value*, which is 9.

3. What is the result of division called?

ANS. It is called the *quotient*, which means *how often*.

4. If I *write* the expression, $18 \div 2 = 9$, what is that sign between the 2 and the 9?

ANS. It is the sign of *equality*, and the expression is read, "18 divided by 2 is equal to 9."

NOTE.—The teacher should write upon the blackboard the following and similar expressions: the pupil should read them, and give the quotient.

5. $24 \div 8 =$	9. $45 \div 9 =$	13. $63 \div 7 =$
6. $72 \div 6 =$	10. $96 \div 12 =$	14. $56 \div 8 =$
7. $108 \div 9 =$	11. $44 \div 11 =$	15. $132 \div 11 =$
8. $84 \div 12 =$	12. $48 \div 6 =$	16. $45 \div 5 =$

17. How many 4's are there in 25?

MODEL OF ANSWER.—Six 4's and a remainder 1.

18. How many 3's are there in 16? 17? 19? 20? 22? 23? 25? 26? 28? 29? 31? 32?

19. How many 4's are there in 33? 34? 47?

20. How many 5's are there in 26? 37? 48?

21. How many 6's are there in 39? 40? 53?

22. How many 7's are there in 54? 55? 65?

23. How many 8's are there in 67? 77? 87?

24. How many 9's are there in 49? 61? 80?

25. What is 1-fourth of 84?

ANALYSIS.—84 is 8 *tens* and 4; 1-fourth of 8 *tens* is 2 *tens*, and 1-fourth of 4 is 1; 2 *tens* and 1 are 21: therefore 1-fourth of 84 is 21.

26. What is 1-half of 68? 46? 84? 24?

27. What is 1-third of 39? 63? 66? 99?

28. What is 1-fourth of 48? 88? 80? 40?

29. What is 1-fifth of 50? 55? 100? 105?

30. What is 1-sixth of 60? 66? 120? 126?

31. What is 1-fourth of 92?

ANALYSIS.—92 is 9 *tens* and 2; 1-fourth of 9 *tens* is 2 *tens* and 1 *ten* remains; 1 *ten* and 2 are 12; 1-fourth of 12 is 3; 2 *tens* and 3 are 23: therefore 1-fourth of 92 is 23.

32. What is 1-half of 32? 56? 74? 98?

33. What is 1-third of 45? 54? 72? 87?

34. What is 1-fourth of 56? 68? 76? 96?

35. What is 1-fifth of 65? 75? 85? 95?

36. What is 1-sixth of 78? 84? 90? 96?

37. What is 1-seventh of 84? 91? 98? 105?

38. What is 1-eighth of 96? 120? 136? 144?

39. $12 + 10$ are 2 times what number?

ANALYSIS.— $12 + 10 = 22$; if 22 is 2 times a number, 1-half of 22, which is 11, must be that number. Therefore $12 + 10$ are 2 times 11.

40. What is 11, in reference to 10 and 12?

ANS. 11 is the *mean*, or *average*, of 10 and 12.

41. What is meant by the average of numbers?

ANS. It is that which equals their sum, if taken as many times as there are numbers. It implies that their sum is distributed into as many equal quantities as there are numbers.

42. What is the average of 5, 6, and 10?

ANALYSIS.— $5 + 6 + 10 = 21$, and there are three numbers; therefore their average is 1-third of 21, which is 7.

43. What is the average of 4, 7, and 13? Of 6, 10, and 11? Of 2, 4, 8, and 10? Of 3, 6, 9, and 10?

44. What is the average of 5, 7, 9, and 11? Of 1, 2, 3, 4, and 5? Of 2, 3, 4, 5, and 6?

45. John's age is 6 years, Mary's 9, Harriet's 11, and Jacob's 14; what is the average of their ages?

LESSON VII.

1. If I buy 7 barrels of flour for 50 dollars, how must I sell them a barrel to gain 13 dollars?

2. How much more do 3 horses cost, at 62 dollars a head, than 6 cows, at 27 dollars a head?

3. 144 are how many times 2? 3? 4? 6? 8?

4. If 11 yards of one kind of cloth cost 88 dollars, and 7 yards of another kind cost 84 dollars, what is the difference in the prices per yard?

5. If 7 persons give each 6 dollars, to be shared equally by 3 widows, what does each widow receive?

6. 60 are how many times 2? 3? 4? 5? 6? 10?

7. If 130 men should form into platoons of 7 each, how many platoons would there be, and how many men would be left?

8. Mention the sets of factors that make 72.

9. How many times 12 are 4 times 18?

SOLUTION.—4 times 18 are 72; 12 is contained in 72 six times; therefore 4 times 18 are 6 times 12.

10. How many times 11 are 3 times 33?

11. How many times 10 are 8 times 15?

12. How many times 9 are 3 times 54?

13. How many times 8 are 7 times 16?

14. How many times 7 are 5 times 21?

15. How many times 6 are 9 times 18?

16. How many times 4 are 11 times 12?

17. How many times 5 are 6 times 20?

18. How many times 3 are 8 times 21?

19. How many times 2 are 10 times 17?

20. How many times 5 are 3 more than 11×7 ?

21. How many times 8 are 2 less than 7×6 ?

22. How many times 7 are 5 more than 9×8 ?

23. 6 less than 9×9 are how many times 5?

24. 4 more than 10×6 are how many times 4?

25. 7 less than 2×49 are how many times 7?

26. 8 less than 3×40 are how many times 4?

27. $9 \times 9 =$ how many times $12 \div 4$?

28. $96 \div 4 =$ how many times $84 \div 7$?

29. Sold 2 pigs at 7 dollars each, and 4 others at 4 dollars each; what price did I average?

ANALYSIS.—If 2 pigs brought 2×7 or 14 dollars, and 4 pigs brought 4×4 or 16 dollars, $2 + 4$ or 6 pigs brought $14 + 16$ or 30 dollars, and the average price was 1-sixth of 30, or 5 dollars.

30. Gave 3 dollars apiece for 5 sheep, and 10 dollars apiece for 2 others; what did they average?

31. Bought 2 oranges at 2 cents each, 2 at 3 cents each, and 2 at 4 cents each; what did they average?

32. Four boys are 9 years old, 2 are 10 years old, and 2 are 12 years old; what is their average age?

33. One day 30 were at school, two days 20, and two days 25; what was the average attendance?

34. A boat sailed 2 hours at 3 miles per hour, 3 hours at 4 miles per hour, and 4 hours at 9 miles per hour; what was its average velocity?

35. A merchant bought a 40-gallon barrel of oil at the rate of 4 gallons for 2 dollars, and paid 2 dollars for the barrel, and 1 dollar for freight; 10 gallons having leaked out, he sold the rest at the rate of 1 dollar a gallon, and the barrel for 1 dollar; did he make, or lose, and how much?

36. Gave a knife, which cost me 25 cents, for 2 balls and 5 cents; what average cost were the balls to me?

37. Traded 3 bushels of oats for 5 pounds of butter; what did the butter cost me a pound, if I could have got 25 cents a bushel for the oats?

38. Frank traded 6 marbles for a kite, the kite for a hook and line, the hook and line for a lot of fish, and sold the fish for 12 cents; what average price did Frank get for the marbles?

39. Bought 30 hogs at the rate of 6 dollars for 2; it cost me at the rate of 5 dollars for 3 to fatten them; 5 of them then died, and I sold the rest at the rate of 50 dollars for 5; did I gain, or lose, and how much?

40. How many are 8×16 , less $150 \div 6$?

LESSON VIII.

1. If 6 men chop a certain quantity of wood in 3 days, how long will it take 2 men to do as much?

ANALYSIS.—If it takes 6 men 3 days, it will take one man 6 times 3 days, or 18 days, and it will take 2 men 1 half of 18 days, or 9 days.

2. In how many days can 3 men do as much work as 7 men can do in 6 days?

3. How many days can 4 men live on the provisions which last 6 men 10 days?

4. How long will hay last 7 cows, which would last 9 cows 7 weeks?

5. How long will oats last 8 horses, which would last 12 horses 6 weeks?

6. How long will water last 11 sailors, which would last 5 sailors 11 days?

7. How many men must be employed to do that work in 12 days which would take 8 men 24 days?

8. How many teams must be employed to haul that wheat in 6 days which would take 10 teams 18 days?

9. If 3 yards of muslin cost 30 cents, what will 5 yards of the same cost?

ANALYSIS.—If 3 yards cost 30 cents, 1 yard costs that part of 30 cents which 1 yard is of 3 yards, that is 1 third of 30 cents, which is 10 cents; and 5 yards will cost 5 times 10 cents, or 50 cents.

10. If 4 yards of silk cost 8 dollars, what will 11 yards of the same cost?

11. If 5 sacks of wheat cost 15 dollars, what will 9 sacks cost?

12. If 12 barrels of flour cost 72 dollars, what do 7 barrels cost?

13. If 6 bushels of potatoes cost 192 cents, what will 9 bushels cost at the same price?

14. If 8 weeks' board cost 40 dollars, what would 9 weeks' board cost? 11 weeks? 12 weeks?

15. If a train goes 80 miles in 4 hours, how far would it go in 10 hours, at the same rate?

16. At the rate of 12 miles in 3 hours, how long would a man be in walking 28 miles?

ANALYSIS.—If he walks 12 miles in 3 hours, in 1 hour he walks that part of 12 miles which 1 is of 3, that is, 1 third of 12 miles, or 4 miles. At the rate of 4 miles an hour he will be as many hours in walking 28 miles as 4 is contained times in 28. Since 4 is contained in 28 7 times, he will be 7 hours in walking 28 miles.

17. When coal is sold at the rate of 6 dollars for 2 loads, how many loads will 30 dollars buy?

18. Bought 88 cents' worth of tea, and paid with eggs at the rate of 44 cents for 4 dozen; how many dozen had I to give?

19. At the rate of 4 cents for 3 apples, how many apples will 36 cents buy?

ANALYSIS.—If 4 cents buy 3 apples, 36 cents will buy as many times 3 apples as there are 4's in 36. Since there are nine 4's in 36, 36 cents will buy 9 times 3 apples, or 27 apples.

20. At the rate of 3 peaches for 5 cents, how many peaches will 25 cents buy?

21. If 12 cents buy 7 pears, how many pears can be bought for 60 cents?

22. If 5 bushels of wheat cost 7 dollars, how many bushels will 98 dollars buy?

23. If 6 barrels of cider sell for 10 dollars, how many barrels will 50 dollars buy?

24. If 8 sacks of flour cost 16 dollars, how many sacks can be bought for 160 dollars?

25. If 5 pounds of beef cost 40 cents, how many pounds will 192 cents buy?

26. If it takes 14 men 2 days to mow a field, how long would it take 7 men?

27. If a boat, sailing 45 miles in 3 hours, pursues another which sails 36 miles in 3 hours, and which has had 6 hours' start, in how many hours will the first overtake the second?

28. If 150 dollars are the wages of 5 months, what are the wages of 8 months, and what are the savings, if expenses are 12 dollars a month?

CHAPTER VII.

EXERCISES IN FRACTIONS.

LESSON I.

1. If you cut an apple into 2 equal parts, what is each part called?

ANS. *One half* of the apple.

2. What is meant by one half of any thing?

ANS. One half of any thing is one of two equal parts which make that thing.

3. How many halves, therefore, has any thing?

4. If you cut a pencil into 3 equal pieces, what is each piece called?

ANS. One third of the pencil, and 2 of them are called *two thirds* of the pencil.

5. What is meant by 1 third of any thing?

ANS. One third of a thing is one of 3 equal parts which make that thing; and 2 thirds of a thing are 2 of 3 equal parts which make that thing.

6. How many thirds, therefore, has any thing?

7. If you cut an orange into 4 equal parts, what is each part called?

ANS. *One fourth* or *one quarter* of the orange.

8. What part of the whole orange are 2 of the 4 equal parts? 3 of them?

9. What is meant by 1 fourth of any thing? 2 fourths? 3 fourths? 3 quarters?

10. How many fourths or quarters has any thing?

11. If a melon is cut into 5 equal pieces, what is 1

of the pieces called? What are 2 of them called? 3 of them? 4 of them?

12. What is meant by 1 fifth of any thing?

13. How many fifths has any thing?

14. If a pie is cut into 6 equal pieces what part of the pie is each piece? What are 2 of them? 5?

15. What is meant by 5 sixths of any thing?

16. How many sixths has any thing?

17. What is meant by sevenths of any thing? eighths? ninths? tenths? elevenths?

18. How many sevenths has any thing? eighths? ninths? tenths? elevenths? twelfths?

19. If any thing is divided into 21 equal parts, what is each part called?

ANS. *One twenty-first* part of that thing.

20. What is 1 of 22 equal parts which make a thing?

ANS. *One twenty-second* part of that thing.

21. What is meant by 5 twenty-thirds of any thing?

22. How many halves have 2 things?

ANALYSIS.—If 1 thing has 2 halves, 2 things must have 2 times 2 halves, or 4 halves. Therefore, 2 things have 4 halves.

23. How many halves have 3 things? 4 things? 5? 6? 7? 8? 9? 10? 11? 12?

24. How many thirds have 2 things? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14?

25. How many fourths have 2 things? 3? 4? 5? 6? 7? 8? 9? 10? 15? 20? 25? 30?

26. How many fifths have 7 things? 8 things?

27. How many sixths have 9 things? 11 things?

28. How many sevenths have 5 things? 15 things?

29. How many whole bushels in 6 half bushels?

ANALYSIS.—Since 2 half bushels are 1 bushel, 6 half bushels are as many bushels as there are 2's in 6, that is, 3 bushels.

30. How many wholes in 8 halves? 18 halves? 22 halves? 6 thirds? 15 thirds? 21 thirds? 8 fourths? 32 fourths? 48 fourths? 72 fourths? 15 fifths? 35 fifths? 24 sixths? 54 sixths? 42 sevenths? 56 eighths? 72 ninths? 70 tenths?

LESSON II.

1. What part of 2 cents is 1 cent?

ANALYSIS.—If 2 cts. were divided into parts of 1 ct. each, there would be 2 such parts; therefore 1 ct. is 1-half of 2 cts.

2. One ct. is what part of 3 cts.? Of 4 cts.? Of 5 cts.? Of 6 cts.? Of 7 cts.? Of 8 cts.? Of 9 cts.? Of 10 cts.? Of 11 cts.? Of 12 cts.?

3. Three cents are what part of 6 cts.? Of 9? Of 12? Of 15? Of 18? Of 21? Of 24?

4. If you had twenty marbles, how would you find 1-fourth of them? 1-fifth of them?

5. How can you find what is 1-fifth of any abstract number? 1-sixth? 1-seventh? 1-eighth? 1-ninth? 1-tenth? 1-eleventh? 1-twelfth?

6. Milo has 12 cents, and Ernest has 2-thirds as many; how many cents has Ernest?

ANALYSIS.—One-third of 12 cts. is 4 cts., and 2-thirds are 2 times 4 cts., or 8 cts.; therefore Ernest has 8 cents.

7. Thomas has 3-fourths as many cents as Ernest; how many has Thomas?

8. What is 3-fourths of 20? 24? 28? 36?

9. What is 2-thirds of 21? 24? 27? 36?

10. If you had a pencil 3 times as long as your little finger, how could you find the place to cut off 2-thirds of the pencil?

11. How could you find where to cut off 3-fourths of a ribbon, 4 times as long as your finger?

12. If a yard of linen costs 48 cents, what does 3-fourths of it cost? 2-thirds of it? 1-half of it?

13. A boy had 42 cents and spent 2-thirds of the money; how much had he left?

14. If a book costs 40 cents, and a slate 1-half as much, what do both cost?

15. If the price of a horse is 96 dollars, and that of a cow is 1-fourth as much, what is the difference in their prices? What would both cost?

16. If 2 equal oranges cost 6 cents, what part of 6 cents, and how much, does 1 orange cost?

ANALYSIS.—If 2 oranges of equal price cost 6 cts., 1 costs that part of 6 cts. which 1 is of 2, namely, 1-half of 6 cts., which is 3 cts.

17. If 8 cents are equally distributed among 4 boys, what part of 8 cents, and how many, has each?

18. If 3 boys share a cake equally, what part of the cake does each receive? What part do 2 receive?

19. If 3 men of equal wages earn 45 dollars, what part of 45 dollars, and how much, does 1 man receive? What part, and how much do 2 receive?

20. If 4 quarts of syrup cost 32 cts., what part of 32 cts., and how much, does 1 quart cost? 2? 3?

21. At 4 dollars a cord, what part of a cord of wood will 1 dollar buy? 2 dollars? 3? 4?

22. If each of 9 boys receives 1-fourth of a pound of raisins, how many pounds do all receive?

ANALYSIS.—If each receives 1-fourth of a pound, 9 boys receive 9 times 1-fourth, or, 9-fourths of a pound: 9-fourths of a pound are as many pounds as there are 4's in 9, or 2 pounds and 1-fourth.

23. If each of 15 persons gives 1-fourth of a dollar, how many dollars are given by all?

24. If each of 36 persons spends 1-third of a dollar, how many dollars are spent by all?

25. If each of 25 persons receives 1-third of a sack of flour, how many sacks do all receive?

26. If each soldier receives 2-thirds of a pound of bread a day, how many pounds supply 37 soldiers?

27. If each sailor receives 3-fourths of a quart of water a day, how many quarts supply 21 sailors?

28. If each horse eats 3-fourths of a peck of oats a day, how many pecks do 12 horses eat?

29. A peck is 1-fourth of a bushel; how many bushels of oats would the same 12 horses eat per day?

30. At 2-thirds of a mile per hour, how far would the water of a river run in 24 hours?

31. At 3-quarters of a dollar per day, what are the wages of 6 men 6 days?

LESSON III.

1. What is $\frac{1}{6}$ of 60? $\frac{1}{5}$? $\frac{1}{4}$? $\frac{1}{3}$? $\frac{1}{2}$? $\frac{1}{10}$? $\frac{1}{12}$?

2. What is $\frac{1}{5}$ of 30? $\frac{2}{5}$? $\frac{3}{5}$?

3. What is $\frac{1}{6}$ of 30? $\frac{2}{6}$? $\frac{3}{6}$? $\frac{4}{6}$? $\frac{5}{6}$?

4. A's age is 25, and B's is $\frac{4}{5}$ as much; what is the sum of their ages? The difference?

5. How many fifths of an acre are in 9 acres?

6. How many acres are in $\frac{43}{6}$ of an acre?

7. Into how many lots, each $\frac{1}{6}$ of an acre, could you divide 10 acres?

8. If 5 heirs share equally an estate of 55 thousand dollars, what part of it and how much is received by 1 of them? 2? 3? 4?

9. David is 15 years old, and his uncle is $\frac{11}{5}$ as old; how many years older than he, is his uncle?

10. Norman is 18 years old, and his father is 17 years more than $\frac{12}{6}$ as old; how old is his father?

11. Clifton's age is 20, and his grandfather is 20 years more than $\frac{11}{5}$ as old; how old is his grandfather?

12. In a battle $\frac{6}{7}$ of a company survived and 9 were killed; how many were in the company?

13. A horse, which cost 120 dollars, was sold for $\frac{5}{8}$ of the cost; what was the loss?

14. A house which cost 14 hundred dollars, sold for $\frac{9}{10}$ of the cost; what was the gain?

15. Of the trees in an orchard $\frac{2}{8}$ died, and 24 survived; how many trees had the orchard?

16. If 1 day in seven is a Sabbath, how many years of Sabbaths has a person had, who is 70 years old?

17. If $\frac{1}{3}$ of a person's time, on the average, has been taken for sleep, how many years of sleep has a person 81 years old taken? How many years has such a person been awake?

18. A person having 40 acres of land, sold $\frac{5}{8}$

of it, and bought again 1-fifth as much as he sold; how many acres had he then?

19. If a yard of cloth costs 6-sevenths of 14 dollars, how many yards will 6-eighths of 64 dollars buy?

20. How many times 4 are 2-thirds of 36?

21. How many times 5 are 3-fourths of 60?

22. How many times 6 are 4-fifths of 60?

23. How many times 7 are 5-sixths of 84?

24. How many times 8 are 6-sevenths of 112?

25. How many times 9 are 7-eighths of 72?

26. How many times 3 are 2-thirds of 27?

27. How many times 2 are 3-fifths of 40?

28. How many times 10 are 5-sevenths of 56?

29. How many times 11 are 4 plus 7-ninths of 108?

30. How many times 12 are 3 less than 7-tenths of 90?

31. How many are 3 times 16, plus 1-eighth of 16?

32. How many are 2 times 12, plus 3-fourths of 12?

33. How many are 4 times 18, plus 5-ninths of 18?

34. How many are 5 times 24, plus 5-eighths of 24?

35. How many are 6 times 9, plus 7-ninths of 9?

36. How many dollars in 50-tenths of a dollar?

37. After 3-tenths of a school had left, 28 remained; what was the number at school?

38. Having 100 cents, I spent 2-tenths of them, then 4-fifths of the remainder; how much had I left?

39. Of 99 cents I spent 1-eleventh, then 1-tenth of the remainder, then 1-ninth, 1-eighth, 1-seventh, 1-sixth, 1-fifth, 1-fourth, 1-third, and 1-half of the successive remainders respectively; what remained at last?

40. Ten mills make 1 cent; what part of a cent are 9 mills? 8 mills? 7 mills? 6 mills? 3 mills?

41. Ten cents make 1 dime; how much is a half-dime?

42. Ten dimes make 1 dollar; how many dimes in 5 dollars and 4-tenths of a dollar?

43. Ten dollars make 1 eagle; how many dollars in 1 double-eagle, 2 eagles, and 3 half-eagles?

44. How many feet length in 72-twelfths of a foot?

LESSON IV.

1. Of what number is 7 one-half?

ANALYSIS.—Since a number has 2 halves, if 7 is 1-half of a number, 2 times 7, or 14, must be that number. Therefore 7 is 1-half of 14.

2. Of what is 8 one-third? One-fourth?

3. Of what is 15 one-fifth? One-sixth?

4. Of what is 12 one-seventh? One-eighth?

5. Of what is 11 one-ninth? One-tenth?

6. Of what is 20 one-eleventh? One-twelfth?

7. A merchant lost 10 thousand dollars, which was 1-fourth of his property; how much had he left?

8. A raised 20 bushels of wheat, which was 1-fifth of B's wheat crop; how much wheat did B raise?

9. A man saved 12 dollars a month, which was 1-sixth of his income; what was his income?

10. Walter spent 14 cents, which was 1-seventh of what he had; how much money had he?

11. Of 3 times what number is 5 one-sixth?

ANALYSIS.—If 5 is 1-sixth of 3 times a number, 6 times 5, or 30, must be 3 times that number. If 30 is 3 times a number, 1-third of 30, or 10, must be that number. Therefore 5 is 1-sixth of 3 times 10.

12. Of 4 times what number is 6 one-twelfth?

13. Of 5 times what number is 7 one-tenth?

14. Of 6 times what number is 8 one-ninth?

15. Of 7 times what number is 7 one-ninth?

16. Of 8 times what number is 16 one-sixth?

17. Of 1-third of what number is 4 one-half?

ANALYSIS.—If 4 is 1-half of 1-third of a number, 2 times 4, or 8, must be 1-third of that number. If 8 is 1-third of the number, 3 times 8, or 24, must be the number. Therefore 4 is 1-half of 1-third of 24.

18. Of 1-fourth of what number is 5 one-third?

19. Of 1-fifth of what number is 6 one-half?

20. Of 1-sixth of what number is 7 one-half?

21. Of 1-seventh of what number is 3 one-fourth?

22. Of $\frac{1}{8}$ of what number is 2 $\frac{1}{5}$?
 23. Of $\frac{1}{9}$ of what number is 8 $\frac{1}{10}$?
 24. A man has 5 cows, which are $\frac{1}{4}$ as many as $\frac{1}{5}$ of his sheep; how many sheep has he?
 25. John's age is 10 years, which is $\frac{1}{7}$ of twice his father's age; how old is his father?
 26. A's share in an oil-well was $\frac{1}{10}$, and he sold $\frac{1}{2}$ of his share for 5 hundred dollars; what was the value of the whole well, at that rate?
 27. B's share in a mill was $\frac{1}{12}$, and he sold $\frac{1}{3}$ of his share for 2 thousand dollars; what was the value of the mill, at that rate?
 28. C's share in a foundry was $\frac{1}{3}$, and he sold $\frac{1}{5}$ of his share for 4 thousand dollars; what was the value of the foundry, at that rate?
 29. D's share in a store was $\frac{1}{8}$, and he sold $\frac{1}{2}$ of his share for 1 thousand dollars; what was the value of the store, at that rate?
 30. Of what number is 8 $\frac{2}{3}$?
- ANALYSIS.—If 8 is $\frac{2}{3}$ of a number, $\frac{1}{2}$ of 8, or 4, must be $\frac{1}{3}$ of that number; and if 4 is $\frac{1}{3}$ of the number, 3 times 4, or 12, must be the number. Therefore 8 is $\frac{2}{3}$ of 12.
31. Of what number is 10 $\frac{2}{3}$?
 32. Of what number is 12 $\frac{2}{3}$?
 33. Of what number is 12 $\frac{3}{4}$?
 34. Of what number is 18 $\frac{3}{4}$?
 35. Of what number is 12 $\frac{4}{5}$?
 36. Of what number is 9 $\frac{3}{5}$?
 37. Of what number is 10 $\frac{2}{5}$?
 38. Of what number is 30 $\frac{5}{6}$? $\frac{3}{4}$? $\frac{3}{5}$? $\frac{2}{7}$? $\frac{6}{7}$?
 39. Of what number is 60 $\frac{2}{3}$? $\frac{2}{5}$? $\frac{2}{7}$? $\frac{2}{9}$? $\frac{3}{5}$? $\frac{4}{7}$? $\frac{5}{8}$? $\frac{6}{9}$? $\frac{10}{11}$?
 40. Of what is 48 $\frac{2}{3}$? $\frac{3}{4}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{6}{7}$? $\frac{8}{12}$?
 41. Of what is 45 $\frac{3}{8}$? $\frac{5}{6}$? $\frac{9}{11}$? $\frac{9}{10}$?

LESSON V.

1. Two-thirds of 18 are 3-fourths of what number?

ANALYSIS.—1-third of 18 is 6; 2-thirds of 18 must be 2 times 6, or 12. If 12 is 3-fourths of a number, 1-third of 12, or 4, must be 1-fourth of that number; and if 4 is 1-fourth of the number, 4 times 4, or 16, must be the number. Therefore 2-thirds of 18 are 3-fourths of 16.

2. Two-thirds of 21 are 7-ninths of what number?
3. Two-fifths of 40 are 4-sevenths of what number?
4. Three-eighths of 56 are 3-sevenths of what number?
5. Four-sevenths of 63 are 3-fifths of what number?
6. Five-ninths of 54 are 5-sixths of what number?
7. Six-tenths of 70 are 3-fifths of what number?
8. Six-eighths of 72 are 3-fourths of what number?
9. Eight-tenths of 60 are 4-fifths of what number?
10. A man, having lost 6 thousand dollars, which was 3-eighths of his property, found that the remainder was 5-twelfths of his neighbor's property; what was the value of his neighbor's property?
11. If 1-half of a barrel of flour costs 4 dollars, what does a whole barrel cost?

ANALYSIS.—If 1-half of a barrel of flour costs 4 dollars, 2 halves, or a whole barrel, cost 2 times 4 dollars, or 8 dollars.

12. If 1-half of a pound of sugar costs 5 cents, how much does 1 pound cost? 2 pounds? 7 pounds?
13. If 1-third of a yard of cloth costs 12 cents, how much does 1 yard cost? 2 yards? 10 yards?
14. At the rate of 1-fourth of an acre for 15 dollars, what costs 1 acre of land? 5 acres? 9 acres?
15. If 1-fifth of a pound of confectionery costs 4 cents, what costs 1 pound? 4 pounds?
16. If 1-sixth of a ton of iron costs 7 dollars, how much does 1 ton cost? 2 tons? 6 tons?
17. If 1-seventh of a man's income is 2 hundred dollars, what is his income?

18. If $\frac{1}{8}$ of a bushel of apples costs 10 cents, what costs 1 bushel? 2 bushels? 8 bushels?

19. If $\frac{2}{3}$ of a barrel of pork cost 12 dollars, what does 1 barrel cost?

ANALYSIS.—If $\frac{2}{3}$ of a barrel cost 12 dollars, $\frac{1}{3}$ costs $\frac{1}{2}$ of 12 dollars, which is 6 dollars, and $\frac{3}{3}$, or the whole barrel, will cost 3 times 6 dollars, or 18 dollars.

20. If $\frac{2}{3}$ of a barrel of beef cost 10 dollars, what does 1 barrel cost? 2 barrels? 3 barrels?

21. If $\frac{3}{4}$ of a yard of cloth cost 9 dollars, what costs 1 yard? 2 yards? 10 yards?

22. If $\frac{3}{5}$ of an acre of land cost 18 dollars, what will $\frac{5}{6}$ of an acre cost?

ANALYSIS.—If $\frac{3}{5}$ of an acre cost 18 dollars, $\frac{1}{5}$ costs $\frac{1}{3}$ of 18 dollars, which is 6 dollars, and $\frac{5}{5}$, or 1 acre, will cost 5 times 6 dollars, or 30 dollars. If 1 acre costs 30 dollars, $\frac{5}{6}$ of an acre cost $\frac{5}{6}$ of 30 dollars: $\frac{1}{6}$ of 30 is 5, and $\frac{5}{6}$ are 5 times 5, or 25. Therefore if $\frac{3}{5}$ of an acre cost 18 dollars, $\frac{5}{6}$ of it cost 25 dollars.

23. Gave 32 dollars for $\frac{2}{3}$ of an acre; if I had bought $\frac{3}{4}$ of an acre at the same rate, what would have been the cost?

24. Gave 36 cents for $\frac{6}{7}$ of a yard of linen; at that rate what would $\frac{5}{6}$ of a yard cost?

25. Gave 80 cents for $\frac{5}{8}$ of a gallon of wine; at that rate what would $\frac{3}{4}$ of a gallon cost?

26. Gave 6 dollars for $\frac{3}{4}$ of a ton of coal; at that rate what would 2 and $\frac{1}{4}$ tons cost?

27. In a school were 24 girls; $\frac{5}{6}$ of the number of girls were $\frac{2}{3}$ of the number of boys; what was the whole number of pupils in that school?

28. A pole had $\frac{1}{10}$ of its length set in the earth; $\frac{1}{3}$ of the part above the earth broke off and left 36 feet of that part standing; how long was the pole?

29. If 72 is $\frac{8}{11}$ of a number, what is $\frac{2}{3}$ of it? $\frac{5}{11}$ of it?

30. Lost $\frac{1}{10}$ of my money; then made $\frac{1}{9}$ of the remainder; had I less, or more, than at first?

LESSON VI.

1. How may parts of quantities be expressed in figures, instead of words?

ANS. By placing below a line a figure or figures expressing the *name* of the parts, and above the line a figure or figures expressing the intended *number* of such parts.

2. Explain the expression 2 over 3, written $\frac{2}{3}$.*

ANS. The 3 means that a unit is supposed to be divided into 3 equal parts, or *thirds*, and the 2 means that 2 of these thirds are expressed: the whole expression, therefore, is read *two-thirds*.

3. Explain $\frac{1}{2} : \frac{3}{4} : \frac{3}{5} : \frac{4}{5} : \frac{4}{7} : \frac{5}{8} : \frac{5}{8} : \frac{6}{7} : \frac{6}{8} : \frac{7}{8} : \frac{7}{10} :$
 $\frac{8}{20} : \frac{9}{17} : \frac{10}{21} : \frac{11}{32} : \frac{16}{23} : \frac{17}{45} : \frac{56}{94} : \frac{101}{201} : \frac{125}{126} : \frac{253}{467} : \frac{572}{622} :$
 $\frac{603}{604} : \frac{750}{850} : \frac{888}{999} : \frac{939}{1001}.$

4. How would you express in figures *five-sevenths*? *Twelve thirty-firsts*? *Twenty forty-seconds*? *Thirty fifty-thirds*? *Seventy-six one-hundred-and-firsts*? *Eighty-nine two-hundred-and-seconds*?

5. What are such numbers called?

ANS. They are called *fractions*, a name derived from a Latin word signifying *broken*.

6. Why are such numbers called *fractions*?

ANS. Because they represent a unit as *broken*, or divided into parts.

7. What, then, is a fraction?

ANS. A fraction is a number expressing one or more of the parts of a unit.

8. What is the number below the line called?

ANS. The *denominator*, because it *names* the parts into which the unit is supposed to be divided.

9. What is the number above the line called?

ANS. The *numerator*, because it states the *number* of parts expressed by the fraction.

* Such expressions should be explained from the blackboard.

10. How many times 1 are $\frac{6}{2}$?

ANALYSIS.— $\frac{6}{2}$ are as many times 1 as $\frac{6}{2}$ contain $\frac{2}{2}$ which make

1. Since 6 halves contain 2 halves 3 times, $\frac{6}{2}$ are 3 times 1, or 3.

OR, Because 2 halves are 1, 6 halves are as many 1's as there are 2's in 6. Since 6 is three 2's, 6 halves are three 1's, or 3.

11. How can we find the number of units expressed by a fraction, more quickly than by such reasoning?

ANS. By dividing the numerator by the denominator.

12. How many times 1 are $\frac{4}{2}$? $\frac{8}{2}$? $\frac{10}{2}$? $\frac{16}{2}$? $\frac{20}{2}$?

13. How many times 1 are $\frac{3}{3}$? $\frac{6}{3}$? $\frac{18}{3}$? $\frac{24}{3}$? $\frac{36}{3}$?

14. How many times 1 are $\frac{4}{4}$? $\frac{12}{4}$? $\frac{20}{4}$? $\frac{36}{4}$? $\frac{44}{4}$?

15. How many times 1 are $\frac{5}{5}$? $\frac{20}{5}$? $\frac{35}{5}$? $\frac{50}{5}$? $\frac{60}{5}$?

16. How many times 1 are $\frac{6}{6}$? $\frac{12}{6}$? $\frac{30}{6}$? $\frac{42}{6}$? $\frac{48}{6}$?

17. How many times 1 are $\frac{7}{7}$? $\frac{21}{7}$? $\frac{28}{7}$? $\frac{42}{7}$? $\frac{49}{7}$?

18. How many times 1 are $\frac{23}{4}$?

ANALYSIS.— $\frac{23}{4}$ are as many times 1 as $\frac{23}{4}$ contain $\frac{4}{4}$, which make 1. Since 23-fourths contain 4-fourths 5 times and 3-fourths remain, $\frac{23}{4}$ are 5 times 1, and $\frac{3}{4}$ of 1, or $5\frac{3}{4}$.

19. What is such a number as $5\frac{3}{4}$ called?

ANS. A *mixed* number, because it is composed of a whole number and a fraction.

20. How many times 1 are $\frac{5}{2}$? $\frac{8}{3}$? $\frac{15}{4}$? $\frac{29}{5}$? $\frac{43}{6}$?

21. How many times 1 are $\frac{47}{6}$? $\frac{55}{7}$? $\frac{57}{8}$? $\frac{59}{9}$? $\frac{61}{8}$?

22. How many times 1 are $\frac{64}{9}$? $\frac{73}{10}$? $\frac{87}{10}$? $\frac{42}{11}$? $\frac{43}{12}$?

23. What is this process called?

ANS. Finding the value of a fraction.

24. What are those fractions called whose value is either a whole or mixed number?

ANS. They are called *improper* fractions, because the proper expression for their value would be a whole or mixed number.

25. How can an improper fraction be known?

ANS. Its numerator is greater than its denominator.

26. What are those fractions called whose value is less than a unit?

ANS. *Proper* fractions, because their value is properly written as a fraction, since it cannot be expressed as a whole number.

LESSON VII.

1. In $\frac{7}{15}$ what is 15 called? What does it mean?
2. Explain the figures in $\frac{5}{13} : \frac{15}{23} : \frac{9}{4} : \frac{3}{11} : \frac{11}{3}$.
3. What kind of a fraction is $\frac{10}{3}$? Why?
4. What kind of a fraction is $\frac{3}{10}$? Why?
5. What kind of a number is $8\frac{3}{10}$? Why?
6. A foot-rule is marked off into 12 equal parts, called inches; how can you find $\frac{7}{12}$ of a foot?
7. A yard-stick is marked off into 3 equal parts, called feet; how can you find $\frac{2}{3}$ of its length?
8. Reduce $1\frac{5}{3}$ to a whole number.
9. Reduce $1\frac{6}{3}$ to a mixed number.
10. Can you reduce $\frac{3}{8}$ to a whole or mixed number?
11. What kind of number must be used to express five-sixths with figures?
12. What kind of number would be the most proper to express the value of *twelve-fourths*?
13. What fraction expresses *one-half* of 1?
14. What is $\frac{1}{2}$ of 3?

ANALYSIS.—If $\frac{1}{2}$ of 1 is $\frac{1}{2}$, $\frac{1}{2}$ of 3 must be 3 times $\frac{1}{2}$, or $\frac{3}{2}$, which is $\frac{3}{2}$ of 1.

OR, Since in 1 there are 2 halves of 1, in 3 there are 3 times 2 halves of 1, or 6 halves of 1, and in $\frac{1}{2}$ of 3 there must be as many halves of 1 as are contained twice in 6 halves of 1, that is, 3 halves of 1; therefore $\frac{1}{2}$ of 3 is $\frac{3}{2}$ of 1.

15. What is $\frac{1}{2}$ of 5? $\frac{1}{3}$ of 4? $\frac{1}{4}$ of 7? $\frac{1}{5}$ of 4? $\frac{1}{6}$ of 11? $\frac{1}{7}$ of 5? $\frac{1}{8}$ of 13? $\frac{1}{9}$ of 8? $\frac{1}{10}$ of 17? $\frac{1}{11}$ of 12?

16. What two equivalent meanings has the fraction 4 over 2? 7 over 12? $\frac{25}{30}$? $\frac{43}{65}$? $\frac{24}{6}$? $\frac{39}{4}$? $\frac{17}{18}$?

17. John saved from his wages 3 half-dollars, and received 2 half-dollars more from the sale of his skates; how many dollars had he then?

18. How many times 1 are $\frac{3}{2} + \frac{2}{2}$?

19. The grocer sold me at one time 5 half-bushels of apples, at another 7 half-bushels, and at another 3 half-bushels: how many bushels did he sell me?

20. How many times 1 are $\frac{5}{2} + \frac{7}{2} + \frac{3}{2}$?

21. How many times 1 are $\frac{2}{3} + \frac{4}{3} + \frac{6}{3}$?
22. Bought $\frac{1}{4}$, then $\frac{2}{4}$, then $\frac{3}{4}$, then $\frac{3}{4}$ of a pound of tea; how many pounds in all did I buy?
23. How many times 1 are $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}$?
24. A weaver wove $\frac{5}{6}$ of a yard of carpet one day, $\frac{2}{6}$ of a yard the next day, and $\frac{1}{6}$ of a yard the next; how many yards of carpet did he weave in those three days?
25. How many times 1 are $\frac{5}{7} + \frac{6}{7} + \frac{9}{7} + \frac{12}{7}$?
26. How many dollars are $\frac{3}{8} + \frac{5}{8} + \frac{7}{8} + \frac{8}{8}$ of a dollar?
27. How many units are $\frac{1}{9} + \frac{2}{9} + \frac{3}{9} + \frac{4}{9} + \frac{5}{9} + \frac{6}{9} + \frac{7}{9}$?
28. Having $\frac{9}{10}$ of a dime, I spent $\frac{4}{10}$ of a dime; how much money had I left?
29. How many units are $\frac{28}{11} - \frac{6}{11}$? $\frac{87}{12} - \frac{3}{12}$?
30. How many are $\frac{47}{10} - \frac{6}{10}$? $\frac{39}{9} - \frac{5}{9}$? $\frac{63}{8} - \frac{7}{8}$?
31. How many are $\frac{17}{7} - \frac{7}{7}$? $\frac{53}{6} - \frac{4}{6}$? $\frac{62}{5} - \frac{4}{5}$?
32. From a piece of cloth containing $1\frac{3}{4}$ of a yard were cut at one time $\frac{1}{4}$ of a yard, and at another $\frac{1}{4}$ of a yard; how many yards remained in the piece?
33. From a piece of land, containing $\frac{3}{3}$ of an acre, were sold $\frac{7}{3}$ and $\frac{5}{3}$ of an acre; how much remained?
34. From $2\frac{5}{2}$ take the sum of $\frac{9}{2}$ and $\frac{11}{2}$.
35. From the sum of $\frac{9}{10}$ and $\frac{8}{10}$ take $\frac{7}{10}$.
36. How many halves are there in $4\frac{1}{2}$?

ANALYSIS.—Since in 1 there are 2 halves, in 4 there are 4 times 2 halves, or 8 halves; $\frac{8}{2}$ and $\frac{1}{2}$ are $\frac{9}{2}$; therefore there are $\frac{9}{2}$ in $4\frac{1}{2}$.

37. How many times $\frac{1}{2}$ in $8\frac{1}{2}$? 9? $12\frac{1}{2}$? 15?
38. How many thirds in $4\frac{1}{3}$? $5\frac{2}{3}$? 7? $9\frac{1}{3}$? 10?
39. How many times $\frac{1}{3}$ in 11? $11\frac{1}{3}$? 12? $12\frac{2}{3}$?
40. How many times $\frac{1}{4}$ in 7? $8\frac{1}{4}$? 9? $10\frac{3}{4}$?
41. How many times $\frac{1}{5}$ in 3? $4\frac{1}{5}$? $5\frac{2}{5}$? $9\frac{4}{5}$?
42. How many times $\frac{1}{6}$ in 5? $6\frac{1}{6}$? $7\frac{2}{6}$? $11\frac{5}{6}$?
43. How many times $\frac{1}{7}$ in 4? $7\frac{1}{7}$? $8\frac{2}{7}$? $10\frac{5}{7}$?
44. How many times $\frac{1}{8}$ in 2? $5\frac{1}{8}$? $7\frac{3}{8}$? $9\frac{7}{8}$?
45. How many times $\frac{1}{9}$ in 9? $8\frac{1}{9}$? $6\frac{4}{9}$? $5\frac{5}{9}$?
46. How many tenths in 11? $12\frac{3}{10}$? $14\frac{7}{10}$? 15?
47. Reduce 9 to tenths, elevenths, twelfths.
48. Reduce $20\frac{5}{8}$ to an improper fraction.
49. Reduce $10\frac{1}{2}$ to an improper fraction.

LESSON VIII.

1. What cost 5 lemons at $2\frac{1}{2}$ cents apiece?

ANALYSIS.—If 1 lemon costs $2\frac{1}{2}$ cents, 5 lemons will cost 5 times $2\frac{1}{2}$ cents. 5 times 2 cents are 10 cents; 5 times $\frac{1}{2}$ of a cent are $\frac{5}{2}$ of a cent, or $2\frac{1}{2}$ cents; 10 cents and $2\frac{1}{2}$ cents are $12\frac{1}{2}$ cents. Therefore, 5 lemons at $2\frac{1}{2}$ cents apiece cost $12\frac{1}{2}$ cents.

2. What cost 6 apples at $1\frac{1}{2}$ cents apiece?

3. What cost 5 cakes at $\frac{1}{2}$ of a cent each?

4. What will 7 days' board amount to, at $1\frac{1}{3}$ dollars a day? At $1\frac{2}{3}$ dollars a day?

5. What are 8 days' wages at $1\frac{1}{4}$ dollars a day? At $1\frac{3}{4}$ dollars a day?

6. How many are 9 times $2\frac{1}{5}$? $3\frac{2}{5}$? $4\frac{3}{5}$? $5\frac{4}{5}$?

7. What cost 10 bushels of wheat at $\frac{5}{8}$ of a dollar a bushel? At $1\frac{1}{8}$ dollars a bushel?

8. How many are 11 times $6\frac{1}{7}$? $7\frac{2}{7}$? $8\frac{3}{7}$? $9\frac{4}{7}$? $10\frac{5}{7}$? $11\frac{6}{7}$?

9. What cost 7 yards of linen at $\frac{3}{8}$ of a dollar a yard? 5 yards of broadcloth at $6\frac{5}{8}$ dollars a yard?

10. How many are 12 times $8\frac{1}{9}$? $7\frac{2}{9}$? $6\frac{3}{9}$? $15\frac{4}{9}$? $4\frac{5}{9}$? $3\frac{6}{9}$? $12\frac{7}{9}$? $21\frac{8}{9}$?

11. How many dollars will 9 barrels of flour cost at $5\frac{7}{10}$ dollars a barrel? At $4\frac{9}{10}$? At $6\frac{3}{10}$?

12. If $3\frac{3}{4}$ pounds of tea are equally shared by 5 persons, how much does each receive?

ANALYSIS.— $3\frac{3}{4}$ pounds are $\frac{15}{4}$ of a pound. If 5 persons share equally $\frac{15}{4}$ of a pound, each receives 1-fifth of $\frac{15}{4}$, or $\frac{3}{4}$ of a pound.

13. If $4\frac{1}{2}$ dollars are shared equally by 3 persons, how much does each receive?

14. If $5\frac{1}{3}$ yards of cloth are shared equally by 8 persons, how much does each receive?

15. If $6\frac{3}{4}$ bushels of corn are divided equally among 9 persons, how much does each receive?

16. If $7\frac{1}{5}$ dollars are equally divided among 9 persons, how much does each receive? How much do 2 of them receive? 3? 4? 5? 6? 7? 8?

17. If $5\frac{5}{8}$ tons' weight rest equally on 7 supports, how much does each support bear? What do 5 bear?

18. If $6\frac{6}{7}$ tons of coal are equally divided among 12 persons, how much does each receive? How much do 7 of them receive? 8? 9? 10? 11?

19. What is the quotient of $4\frac{3}{8} \div 7$?

20. What is the quotient of $3\frac{5}{9} \div 8$?

21. What is the quotient of $7\frac{7}{10} \div 7$?

22. What is the quotient of $5\frac{5}{11} \div 10$?

23. What is the quotient of $2\frac{1}{12} \div 5$?

24. At $\frac{1}{2}$ of a cent each, how many apples can you buy for $12\frac{1}{2}$ cents?

ANALYSIS.—At $\frac{1}{2}$ of a cent each, $12\frac{1}{2}$ cents will buy as many apples as $\frac{1}{2}$ is contained times in $12\frac{1}{2}$. $12\frac{1}{2}$ is $2\frac{5}{2}$, and $\frac{1}{2}$ is contained in $2\frac{5}{2}$ 25 times; therefore, $12\frac{1}{2}$ cents will buy 25 apples at $\frac{1}{2}$ of a cent each.

25. At $\frac{1}{2}$ of a dollar each, how many caps can be bought for $37\frac{1}{2}$ dollars?

26. At $\frac{2}{3}$ of a cent each, how many sheets of paper can you buy for 12 cents? For $33\frac{1}{3}$ cents?

27. At $\frac{3}{4}$ of a dollar apiece, how many bottles of wine can be bought for 12 dollars? For $20\frac{1}{4}$ dollars?

28. At $\frac{2}{5}$ of a dollar apiece, how many knives can be bought for $9\frac{3}{5}$ dollars? For $10\frac{4}{5}$ dollars?

29. If each support bears $\frac{5}{8}$ of a ton, how many supports are needed for $7\frac{3}{8}$ tons? For $9\frac{1}{8}$ tons?

30. How many persons will $5\frac{5}{7}$ tons of coal supply, if each receives $\frac{4}{7}$ of a ton?

31. At $\frac{5}{8}$ of a dollar a yard, how many yards of cloth will 10 dollars buy? $12\frac{4}{8}$ dollars? $15\frac{5}{8}$ dollars?

32. At $\frac{1}{9}$ of a dollar a dozen, how many dozen of eggs can be bought for 5 dollars? $5\frac{5}{9}$ dollars?

33. How many times $\frac{7}{10}$ in $8\frac{4}{10}$?

34. How many times $\frac{4}{11}$ in $8\frac{8}{11}$?

35. How many times $\frac{5}{12}$ in $5\frac{5}{12}$?

36. What is the quotient of $10 \div \frac{2}{3}$? $10\frac{2}{3} \div \frac{2}{3}$?

37. What is the quotient of $5 \times 2\frac{1}{7} \div \frac{3}{7}$?

38. What is the quotient of $3 \times 4\frac{7}{8} \div 3$?

39. What is the value of $7 \times 3\frac{5}{9} \div \frac{2}{9}$?

LESSON IX.

1. How many sixths are there in $\frac{2}{3}$ of 1?

ANALYSIS.—Since there are $\frac{6}{6}$ in 1, $\frac{1}{3}$ of 1 is $\frac{1}{3}$ of $\frac{6}{6}$, which is $\frac{2}{6}$, and $\frac{2}{3}$ of 1 must be 2 times $\frac{2}{6}$, or $\frac{4}{6}$. Therefore, there are $\frac{4}{6}$ in $\frac{2}{3}$ of 1.

2. Comparing $\frac{2}{3}$ with $\frac{4}{6}$, what do you notice?

ANS. That $\frac{2}{3}$ becomes $\frac{4}{6}$ by multiplying both parts or terms by 2.

3. If $\frac{2}{3} = \frac{4}{6}$, what does it show?

ANS. It shows that multiplying both terms of a fraction by the same number does not affect its value.

4. How many ninths are there in $\frac{2}{3}$ of 1?

5. By what number would you multiply both terms of $\frac{2}{3}$ to reduce it to ninths?

6. Show by analysis how many twelfths there are in $\frac{3}{4}$. Produce the same by multiplication.

7. How many sixteenths in $\frac{3}{4}$? $\frac{5}{8}$? $\frac{7}{8}$?

8. How many eighteenthths in $\frac{2}{3}$? $\frac{5}{6}$? $\frac{5}{9}$? $\frac{8}{9}$?

9. How many twentieths in $\frac{1}{2}$? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{7}{10}$?

10. How many twenty-firsts in $\frac{2}{3}$? $\frac{3}{7}$? $\frac{4}{7}$? $\frac{5}{7}$? $\frac{6}{7}$?

11. How many seventy-seconds in $\frac{1}{2}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{5}{9}$? $\frac{8}{9}$? $\frac{7}{12}$? $\frac{11}{12}$?

12. How many fiftieths in $\frac{3}{10}$? $\frac{7}{10}$? $\frac{9}{10}$?

13. How many twenty-seconds in $\frac{2}{11}$? $\frac{5}{11}$? $\frac{7}{11}$? $\frac{9}{11}$?

14. What is such a process as converting $\frac{9}{11}$ to twenty-seconds called?

ANS. Reducing a fraction to higher terms.

15. Reduce, by analysis, $\frac{16}{25}$ to hundredths. Do the same by multiplication.

16. How do you find the proper multiplier to produce such a reduction? Construct a rule out of these facts.

17. What multiplier will reduce 20ths to 100ths?

18. What multiplier will reduce 15ths to 90ths?

19. What multiplier will reduce 14ths to 98ths?

20. What multiplier will reduce 13ths to 104ths?

21. What will convert 19ths into 76ths?

22. How many fifths are there in $\frac{1\frac{3}{5}}{1\frac{3}{5}}$ of 1?

ANALYSIS.—Since there are $\frac{1\frac{3}{5}}{1\frac{3}{5}}$ in 1, in $\frac{1}{5}$ of 1 there is $\frac{1}{5}$ of $\frac{1\frac{3}{5}}{1\frac{3}{5}}$, or $\frac{3}{15}$. If $\frac{3}{15}$ are 1-fifth of 1, $\frac{1\frac{3}{5}}{1\frac{3}{5}}$ are as many fifths of 1 as 3 is contained times in 12, that is $\frac{4}{5}$ of 1. Therefore, $\frac{1\frac{3}{5}}{1\frac{3}{5}} = \frac{4}{5}$.

23. Comparing $\frac{1\frac{3}{5}}{1\frac{3}{5}}$ and $\frac{4}{5}$, what do you notice?

ANS. That $\frac{1\frac{3}{5}}{1\frac{3}{5}}$ becomes $\frac{4}{5}$ by dividing both parts or terms by 3.

24. If $\frac{1\frac{3}{5}}{1\frac{3}{5}} = \frac{4}{5}$, what does it show?

ANS. It shows that dividing both terms of a fraction by the same number does not affect its value.

25. Show by analysis how many halves there are in $\frac{3\frac{2}{8}}{3\frac{2}{8}}$. Produce the same by division.

26. How many thirds in $\frac{4}{8}$? $\frac{6}{9}$? $\frac{8}{12}$? $\frac{5}{15}$? $\frac{10}{15}$? $\frac{6}{18}$? $\frac{12}{18}$? $\frac{13}{39}$? $\frac{26}{39}$? $\frac{16}{48}$? $\frac{32}{48}$? $\frac{18}{54}$? $\frac{36}{54}$?

27. What divisor will convert $\frac{18}{24}$ into fourths?

28. How do you find this divisor? Construct a rule out of this process.

29. What is this reduction called?

ANS. Reducing a fraction to lower terms.

30. How many 4ths in $\frac{2}{8}$? $\frac{9}{12}$? $\frac{33}{44}$? $\frac{15}{60}$? $\frac{25}{100}$?

31. How many 5ths in $\frac{16}{40}$? $\frac{36}{60}$? $\frac{80}{100}$? $\frac{50}{125}$?

32. How many 6ths in $\frac{35}{42}$? $\frac{45}{54}$? $\frac{15}{90}$?

33. How many 7ths in $\frac{30}{42}$? $\frac{32}{56}$? $\frac{54}{63}$? $\frac{40}{70}$?

34. How many 8ths in $\frac{33}{88}$? $\frac{60}{96}$? $\frac{70}{80}$? $\frac{140}{160}$?

35. How many 9ths in $\frac{24}{54}$? $\frac{35}{45}$? $\frac{42}{63}$?

36. How many 10ths in $\frac{15}{50}$? $\frac{49}{70}$? $\frac{81}{90}$?

37. How many 11ths in $\frac{8}{22}$? $\frac{25}{55}$? $\frac{56}{88}$? $\frac{81}{99}$?

38. How many 12ths in $\frac{25}{60}$? $\frac{49}{84}$? $\frac{81}{108}$? $\frac{121}{132}$?

39. How many 13ths in $\frac{9}{39}$? $\frac{25}{65}$? $\frac{40}{65}$? $\frac{66}{78}$?

40. How many 14ths in $\frac{2}{28}$? $\frac{21}{98}$? $\frac{55}{70}$? $\frac{65}{70}$?

41. How many 21sts in $\frac{6}{63}$? $\frac{12}{84}$? $\frac{20}{105}$?

42. Reduce $\frac{96}{48}$ to halves, 3ds, 4ths, 6ths, 8ths, 12ths, 16ths, and 24ths.

43. Reduce $\frac{120}{24}$ to as many different fractions of lower terms as possible.

44. A man owns $\frac{4}{12}$ of a mill; how many sixths of it has he? How many thirds?

45. A man owns $\frac{3}{4}$ of a mine; how many 100ths has he?

LESSON X.

1. What is a *common divisor* or *common measure* of two or more numbers?

ANS. Any number which is contained as a factor in each of them.

2. Mention all the common divisors of 8, 24, and 72.

3. Mention all the common measures of 48 and 96.

4. Mention all the common measures of 60 and 120.

5. What is the greatest common divisor or measure of two or more numbers?

ANS. It is the greatest number which is contained as a factor in each of them.

6. Find the greatest common measure of 24 and 36.

7. Find the greatest common measure of 32 and 48.

8. Find the greatest common measure of the three numbers 48, 72, and 96.

9. How can $\frac{24}{48}$ be reduced to its lowest terms?

ANS. Either by dividing both terms by their greatest common measure 24, reducing the fraction at once to $\frac{1}{2}$; or, if we cannot easily find the greatest common measure of both terms, we can keep dividing by any common measure, till no such measure remains.

10. Do it by the last-mentioned method.

11. What are the lowest terms of $\frac{9}{18}$? $\frac{8}{18}$? $\frac{6}{18}$?

12. What are the lowest terms of $\frac{12}{36}$? $\frac{25}{75}$? $\frac{40}{50}$?

13. What are the lowest terms of $\frac{25}{100}$? $\frac{72}{96}$? $\frac{32}{64}$?

14. What are the lowest terms of $\frac{16}{96}$? $\frac{90}{108}$? $\frac{70}{98}$?

15. Reduce $\frac{90}{144}$ to its lowest terms.

16. Reduce $\frac{80}{144}$ to its lowest terms.

17. How large is $\frac{1}{6}$ of an apple, compared with $\frac{1}{3}$ of the same? Compared with $\frac{1}{2}$ of the same?

18. How large is $\frac{1}{4}$ of an orange, compared with $\frac{1}{12}$ of the same? Compared with $\frac{1}{16}$ of the same?

19. How large is $\frac{1}{5}$ of any thing, compared with $\frac{1}{10}$ of the same? Compared with $\frac{1}{20}$ of the same?

20. Reduce $\frac{3}{8}$ and $\frac{9}{12}$ to 24ths, and compare them.

21. When are two or more fractions said to have a common denominator?

ANS. When their denominators are alike.

22. In reducing two or more fractions to a common denominator, what number can you select for their denominator?

ANS. Any number which contains each of the denominators as a factor. Thus, 6 can be a common denominator for fractions whose denominators are 2 and 3.

23. Reduce $\frac{1}{2}$ and $\frac{2}{3}$ to sixths. Is 6 the only common denominator to which they can be reduced?

24. What are the three smallest numbers which you can select as the common denominators of $\frac{2}{3}$ and $\frac{3}{4}$?

25. Reduce $\frac{3}{4}$ and $\frac{2}{5}$ to a common denominator.

MODEL OF ANSWER.—I select 20 as the common denominator to which they are to be reduced, because 20 is one of the numbers which contain both the denominators 4 and 5 as factors. Since there are $\frac{20}{4}$ in 1, $\frac{3}{4}$ of 1 must be $\frac{3}{4}$ of $\frac{20}{4}$, or $\frac{15}{20}$, and $\frac{2}{5}$ of 1 must be $\frac{2}{5}$ of $\frac{20}{5}$, or $\frac{8}{20}$. Therefore the fractions, when reduced, are $\frac{15}{20}$ and $\frac{8}{20}$.

26. Reduce $\frac{1}{4}$ and $\frac{5}{6}$ to a common denominator.

27. Reduce $\frac{3}{5}$ and $\frac{2}{3}$ to a common denominator.

28. Reduce $\frac{1}{2}$ and $\frac{4}{5}$ to a common denominator.

29. Reduce $\frac{2}{3}$ and $\frac{5}{8}$ to a common denominator.

30. Reduce $\frac{2}{3}$ and $\frac{5}{9}$ to a common denominator.

31. Reduce $\frac{3}{4}$ and $\frac{7}{9}$ to a common denominator.

32. Reduce $\frac{3}{5}$ and $\frac{7}{9}$ to a common denominator.

33. Reduce $\frac{5}{6}$ and $\frac{7}{9}$ to a common denominator.

34. Reduce $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ to a common denominator.

35. Reduce $\frac{1}{3}$, $\frac{1}{3}$ and $\frac{1}{5}$ to a common denominator.

36. Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$ to a common denominator.

37. Reduce $1\frac{1}{2}$, $1\frac{2}{3}$ and $\frac{1}{7}$ to a common denominator.

38. Reduce $\frac{7}{10}$, $\frac{3}{20}$ and $\frac{5}{8}$ to a common denominator.

39. How could you divide an orange so as to give $\frac{1}{2}$ of it to one person, and $\frac{1}{3}$ of it to another?

40. Into how many parts would you divide any thing to give $\frac{2}{3}$ of it to one person, and $\frac{1}{4}$ to another?

41. Into how many parts would you divide any thing to give $\frac{2}{5}$ of it to one person, and $\frac{3}{7}$ of it to another?

LESSON XI.

1. Can $\frac{1}{2}$ and $\frac{1}{3}$ be added without being reduced to a common denominator?

ANS. They cannot, because halves and thirds are not like parts of a unit. When they are changed to $\frac{3}{6}$ and $\frac{2}{6}$, they both represent sixths, and their sum is $\frac{5}{6}$.

2. What is the sum of $\frac{1}{7} + \frac{2}{7} + \frac{3}{7}$?

3. Why is the sum of $\frac{3}{8} + \frac{2}{5}$ not $\frac{5}{8}$ or $\frac{5}{5}$ or $\frac{5}{13}$?

4. What is the sum of $\frac{3}{8}$ and $\frac{2}{5}$?

MODEL OF ANSWER.—I select 40 as the common denominator to which $\frac{3}{8}$ and $\frac{2}{5}$ are to be reduced: $\frac{3}{8}$ is $\frac{15}{40}$, and $\frac{2}{5}$ is $\frac{16}{40}$; $\frac{15}{40}$ and $\frac{16}{40}$ are $\frac{31}{40}$; therefore the sum of $\frac{3}{8}$ and $\frac{2}{5}$ is $\frac{31}{40}$.

5. What is the sum of $\frac{3}{4}$ and $\frac{1}{5}$? Of $\frac{2}{3}$ and $\frac{1}{5}$?

6. What is the sum of $\frac{1}{3}$ and $\frac{1}{6}$? Of $\frac{2}{3}$ and $\frac{5}{6}$?

7. What is the sum of $\frac{1}{4}$ and $\frac{1}{7}$? Of $\frac{3}{4}$ and $\frac{6}{7}$?

8. What is the sum of $\frac{1}{5}$ and $\frac{1}{8}$? Of $\frac{3}{5}$ and $\frac{5}{8}$?

9. What is the sum of $\frac{2}{5}$ and $\frac{4}{9}$? Of $\frac{5}{6}$ and $\frac{7}{10}$?

10. What is the sum of $8\frac{1}{2}$ and $5\frac{1}{3}$?

SUGGESTION FIRST.— $8\frac{1}{2} = 8\frac{2}{3}$; $5\frac{1}{3} = 5\frac{2}{3}$; $8\frac{2}{3} + 5\frac{2}{3} = 13\frac{4}{3}$.

SUGGESTION SECOND.— $8\frac{1}{2} = 1\frac{7}{2}$; $5\frac{1}{3} = 1\frac{2}{3}$; $1\frac{7}{2} + 1\frac{2}{3} = 2\frac{17}{6} + 1\frac{4}{6} = 3\frac{21}{6} = 13\frac{1}{2}$.

11. What is the sum of $3\frac{1}{4}$ and $4\frac{1}{5}$? Of $2\frac{3}{5}$ and $2\frac{2}{9}$?

12. What is the sum of $1\frac{4}{7}$ and $1\frac{3}{8}$? Of $10\frac{1}{2}$ and $\frac{4}{5}$?

13. What is the sum of $2\frac{1}{6}$ and $1\frac{2}{7}$? Of $6\frac{3}{7}$ and $1\frac{5}{9}$?

14. A man walked $\frac{1}{4}$ of his journey the first day, and $\frac{1}{5}$ of it the second; what part of his journey did he accomplish in the two days? What part remained?

15. Gave $\frac{3}{8}$ of a dollar for tea and $\frac{3}{5}$ of a dollar for coffee; what part of a dollar did both cost?

16. Paid $\frac{1}{5}$ of a dollar for a knife, $\frac{3}{4}$ of a dollar for skates, and $\frac{3}{10}$ of a dollar for a sled; what did all cost?

17. Sold $\frac{1}{2}$ of an acre of land to one man, $2\frac{1}{4}$ acres to another, and $1\frac{2}{3}$ acres to another; how much to all?

18. Walked $4\frac{1}{4}$ hours one day, $3\frac{2}{5}$ hours the second, and $4\frac{5}{8}$ hours the third day; how many hours in all?

19. Bought $2\frac{1}{4}$ tons of coal at one time, $1\frac{5}{6}$ tons at another, and $3\frac{5}{8}$ tons at another; how much in all?

20. From $\frac{3}{4}$ take $\frac{1}{2}$.22. From $\frac{1}{2}$ take $\frac{3}{7}$.24. From $\frac{3}{5}$ take $\frac{1}{3}$.26. From $2\frac{7}{9}$ take $1\frac{1}{3}$.21. From $2\frac{2}{3}$ take $1\frac{1}{4}$.23. From $1\frac{1}{2}$ take $2\frac{2}{5}$.25. From $\frac{5}{6}$ take $\frac{5}{7}$.

SUGGESTION.— $2\frac{7}{9} - 1\frac{1}{3} = 2\frac{7}{9} - 1\frac{3}{9} = \frac{7}{9} - \frac{3}{9} = \frac{4}{9}$. $2 - 1 = 1$. $\frac{7}{9} + 1 = 1\frac{7}{9}$. This method can be used when the fraction to be subtracted is less than the other.

27. From $3\frac{3}{8}$ take $1\frac{1}{4}$.28. From $5\frac{7}{10}$ take $3\frac{3}{5}$.29. From $4\frac{1}{5}$ take $2\frac{1}{4}$.

SUGGESTION.— $4\frac{1}{5} = 2\frac{1}{5}$; $2\frac{1}{4} = \frac{9}{4}$; $\frac{2}{5} - \frac{9}{4} = \frac{8}{20} - \frac{45}{20} = \frac{37}{20} = 1\frac{17}{20}$. This method can be used when the fraction to be subtracted is greater than the other.

30. From $5\frac{1}{2}$ take $2\frac{3}{4}$.31. From $6\frac{2}{7}$ take $2\frac{2}{3}$.32. From 9 take $3\frac{9}{10}$.33. From 8 take $5\frac{7}{12}$.

34. A lady shopping spent $\frac{1}{6}$ of her money at one store, $\frac{2}{5}$ at another, and $\frac{1}{4}$ at another, and had 11 dollars remaining; what part of her money did she spend, what part remained, and how many dollars had she at first?

35. Having $2\frac{1}{2}$ dollars, I received $5\frac{3}{4}$ more, and then paid out $4\frac{9}{10}$ dollars; how much had I left?

36. To raise 100 dollars, A gave $27\frac{1}{8}$ dollars and B $28\frac{3}{10}$ dollars; how much was yet to be raised?

37. How much less than one is $\frac{1}{3} + \frac{3}{10} + \frac{1}{5}$?

38. After a battle $\frac{1}{9}$ of an army were found killed, $\frac{1}{6}$ wounded, and $\frac{1}{8}$ were missing: 12 thousand answered at roll-call; how large was the army at first?

39. From a piece of cloth containing $31\frac{1}{4}$ yards were cut at one time $5\frac{3}{8}$ yards, at another $6\frac{2}{3}$ yards; how many yards remained in the piece?

40. From a cask containing $35\frac{1}{2}$ gallons of wine, were taken at one time $6\frac{3}{4}$ gallons, and at another $5\frac{5}{8}$ gallons; how much remained in the cask?

41. A pole stands $\frac{1}{18}$ of its length in earth, $\frac{1}{4}$ in water, and 65 feet in air; how long is the pole?

42. What does $\frac{4}{9} + \frac{6}{11}$ lack of 1?

43. If $\frac{1}{3}$ of our time is devoted to sleep, $\frac{1}{4}$ to study, $\frac{1}{16}$ to meals, and $\frac{1}{8}$ to exercise. what part remains for other purposes?

LESSON XII.

1. What cost 4 yards of linen at $\frac{3}{8}$ of a dollar per yard?

FIRST ANALYSIS.—If 1 yard costs $\frac{3}{8}$ of a dollar, 4 yards will cost 4 times $\frac{3}{8}$ of a dollar, or $\frac{12}{8}$ of a dollar, equal to $1\frac{4}{8}$, or $1\frac{1}{2}$ dollars. Therefore at $\frac{3}{8}$ of a dollar per yard, 4 yards of linen will cost 1 dollar and 1-half.

SECOND ANALYSIS.—If 1 yard costs $\frac{3}{8}$ of a dollar, 4 yards will cost 3 of those parts of a dollar which are 4 times as large as *eighths*, that is, $\frac{3}{2}$ of a dollar, equal to 1 dollar and 1-half.

2. What do these two methods of analysis show?

ANS. That a fraction can be multiplied by a number, either by multiplying its numerator, or dividing its denominator by that number.

3. What cost 2 pounds of tea at $\frac{5}{6}$ of a dollar per pound? At $\frac{3}{4}$ of a dollar a pound?

4. What cost 3 quires of paper at $\frac{2}{9}$ of a dollar per quire? What cost 9 quires?

5. What cost 4 barrels of flour at $5\frac{7}{8}$ dollars per barrel? At $6\frac{1}{4}$ dollars per barrel?

6. If a man's wages are $1\frac{7}{10}$ dollars per day, how much does he earn in 5 days? In 10 days?

7. If each man mows $2\frac{1}{2}$ acres per day, how much do 3 men mow in a day? 4 men? 6 men?

8. If each man reaps $1\frac{5}{8}$ acres of rye per day, how much do 2 men reap in a day? 3 men? 6 men?

9. At the rate of 16 dozen of eggs for 3 dollars, how much will 2 dozen cost? 4 dozen? 8 dozen?

10. At the rate of 5 yards for 18 dollars, how much will 2 yards of broad-cloth cost? 3 yards? 6? 9?

11. At the rate of 20 lamps for 9 dollars, how much will 2 lamps cost? 4 lamps? 5? 10?

12. At the rate of 36 knives for 13 dollars, how much will 2 knives cost? 3? 4? 6? 9? 12?

13. At the rate of 48 sheets of paper for 25 cents, what cost 2 sheets? 3? 4? 6? 8? 12? 16? 24?

14. How many are 2 times $\frac{5}{8}$?

MODEL OF ANSWER.—2 times $\frac{5}{8}$ are $\frac{5}{4}$, equal to $1\frac{1}{4}$.

15. How many are 2 times $\frac{9}{10}$? $\frac{11}{12}$? $\frac{5}{14}$? $\frac{7}{16}$?

16. How many are 3 times $\frac{13}{15}$? $\frac{11}{12}$? $\frac{10}{17}$? $\frac{29}{30}$?

17. How many are 4 times $\frac{5}{16}$? $\frac{11}{20}$? $\frac{37}{44}$? $\frac{43}{48}$?

18. How many are 5 times $\frac{38}{45}$? $\frac{39}{50}$? $\frac{53}{60}$? $\frac{91}{100}$?

19. How many times 2 are 6 times $\frac{32}{48}$?

20. How many times 2 are 7 times $\frac{54}{63}$?

21. How many times 3 are 8 times $\frac{108}{48}$?

22. How many times 5 are 9 times $\frac{100}{45}$?

23. How many times 9 are 10 times $\frac{126}{20}$?

24. Eleven times $\frac{13}{22}$ is $\frac{1}{2}$ of what number?

25. Twelve times $\frac{5}{36}$ is $\frac{1}{3}$ of what number?

26. Twenty times $\frac{27}{100}$ is $\frac{1}{5}$ of what number?

27. How many bushels of apples in 7 barrels, each holding $2\frac{5}{8}$ bushels?

28. How many bushels of grain in 9 bags, each holding $2\frac{3}{4}$ bushels?

29. Which method of multiplying the fraction is the shorter, in multiplying $4\frac{7}{9}$ by 9?

30. What effect has it upon a fraction to multiply it by its denominator?

31. How many are 2 times $\frac{3}{2}$?

ANS. By one method of multiplication 2 times $\frac{3}{2}$ are $\frac{6}{2}$ or 3; by another method 2 times $\frac{3}{2}$ are $\frac{3}{1}$ or 3.

32. If the denominator of an expression in a fractional form is 1, does it denote the division of the unit into parts?

33. Does such a denominator affect the value of the numerator?

34. How many are 3 times $\frac{3}{3}$? 3 times $\frac{8}{8}$?

35. How many are 4 times $5\frac{1}{4}$? 5 times $6\frac{2}{5}$?

36. How many are 6 times $7\frac{5}{6}$? 7 times $8\frac{3}{7}$?

37. How many are 8 times $9\frac{1}{8}$? 9 times $10\frac{2}{9}$?

38. How many are 10 times $1\frac{1}{10}$? Solve this in as many different ways as you can.

39. If 12 men can mow a field in $1\frac{1}{2}$ days, how long would it take 1 man to mow it?

LESSON XIII.

1. If 4 yards of linen cost $1\frac{2}{8}$ of a dollar, what is the cost of 1 yard?

ANALYSIS.—If 4 yards cost 12-eighths of a dollar, 1 yard costs 1-fourth of 12-eighths, which is $\frac{3}{8}$ of a dollar.

2. If 4 yards of linen cost $1\frac{1}{2}$ dollars, what is the cost of 1 yard?

ANALYSIS.—If 4 yards cost $1\frac{1}{2}$ dollars, that is $\frac{3}{2}$, of a dollar, 1 yard will cost 3 of those parts of a dollar, which are one-fourth as large as halves, that is, $\frac{3}{8}$ of a dollar.

3. What do these two methods of analysis show?

ANS. That a fraction can be divided by a number, either by dividing its numerator or multiplying its denominator by that number.

4. If 2 pounds of tea cost $1\frac{2}{3}$ dollars, what does 1 pound cost?

5. If 3 quires of paper cost $\frac{2}{3}$ of a dollar, what does 1 quire cost?

6. If 4 barrels of flour cost $23\frac{1}{2}$ dollars, what is the cost of 1 barrel?

7. If a man's wages amount to $8\frac{1}{2}$ dollars in 5 days, how much is that per day?

8. If 4 men mow $11\frac{1}{3}$ acres in one day, what is the average per man?

9. If 7 horses eat $4\frac{3}{8}$ bushels of oats per day, what is the average per horse?

10. Gave $5\frac{1}{4}$ dollars for one barrel of flour, $5\frac{3}{8}$ dollars for another, and $6\frac{5}{8}$ dollars for another; what was the average cost?

11. Gave $\frac{1}{10}$ of a dollar for one musk-melon, $\frac{1}{8}$ of a dollar for another, and $\frac{2}{5}$ of a dollar for a watermelon; what was the average cost of the melons?

12. Gave $\frac{1}{2}$ of a dollar a pair for 3 pairs of chickens, $\frac{1}{3}$ of a dollar a pair for 2 pairs, and $\frac{1}{5}$ of a dollar a pair for 2 pairs; what was the average price per pair?

13. Bought at one time 3 dozen of eggs, at the rate

of 5 dozen for a dollar, at another time 4 dozen at the rate of 10 dozen for a dollar; what was the average price per dozen?

14. From a lot containing $17\frac{3}{4}$ acres were sold $5\frac{3}{8}$ acres, and the rest divided into 11 equal house-lots; how much land was in each lot?

15. From a piece of cloth containing $35\frac{5}{8}$ yards were sold $9\frac{1}{2}$ yards, and the rest was made into 7 coats; how much did the coats take on the average?

16. A man took for himself $\frac{4}{9}$ of his crop of corn, and distributed the rest equally among 10 needy families; what part of his corn did each family receive?

17. A man bought at three times respectively $84\frac{1}{2}$, $90\frac{1}{3}$, and $95\frac{1}{6}$ bushels of oats, which kept his 3 horses 12 months; what was the average consumption per month per horse?

18. If a man travels $10\frac{2}{5}$ miles one day, $10\frac{1}{2}$, $12\frac{3}{10}$, and 12 respectively the next three days, what is his average of travel per day?

19. Bought 3 yards of calico at $6\frac{1}{4}$ cents a yard, and 5 yards of ticking at $12\frac{1}{2}$ cents a yard; what did both cost, and what was the average cost per yard?

20. Bought 8 pounds of sugar at the rate of 11 for a dollar, and 6 ounces of spice at the rate of 22 for a dollar; what did both cost?

21. Sowed $7\frac{1}{2}$ acres with wheat, but $2\frac{3}{4}$ acres were destroyed; how many remained to be reaped?

22. How many are 4 times $2\frac{5}{12}$?

23. What is the average of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$?

24. What is $\frac{3}{5}$ divided by 2?

MODEL OF ANSWER.— $\frac{3}{5}$ divided by 2 is $\frac{3}{10}$.

25. What is $\frac{1}{2}$ divided by 2? By 5? By 6?

26. What is $\frac{1}{4}$ divided by 5? By 7? By 10?

27. What is $\frac{2}{5}$ divided by 2? By 4? By 6?

28. In what other form are such questions as these usually put?

ANS. Instead of, "What is $\frac{4}{5}$ divided by 4?" we ask, "What is $\frac{1}{4}$ of $\frac{4}{5}$?"

LESSON XIV.

1. How is $\frac{1}{4}$ of any number found?

Ans. By dividing it by 4.

2. What is $\frac{1}{4}$ of $\frac{4}{5}$?

FIRST ANALYSIS.—One-fourth of 4-fifths is $\frac{1}{5}$. This result may be obtained by dividing the numerator by 4.

SECOND ANALYSIS.—One-fourth of 4-fifths is 4 of those parts of a unit which are one-fourth as large as fifths, that is, $\frac{4}{20}$; therefore $\frac{1}{4}$ of $\frac{4}{5}$ is $\frac{4}{20} = \frac{1}{5}$. This result may be obtained by multiplying the denominator by 4.

THIRD ANALYSIS.— $\frac{1}{4}$ of $\frac{4}{5}$ is $\frac{1}{4}$ of $\frac{16}{20}$, which is $\frac{4}{20}$ or $\frac{1}{5}$.

3. Having $\frac{1}{2}$ of an acre of land, I sold $\frac{1}{2}$ of it; what part of an acre did I sell?

4. A man, owning $\frac{2}{3}$ of a mill, sold $\frac{1}{2}$ of his interest; what part of the whole property did he sell?

5. James, receiving $\frac{3}{4}$ of an orange, gave away $\frac{1}{3}$ of that; what part of the orange did he give away? What part of the orange did he retain?

6. A man gave his pig $\frac{1}{2}$ of $\frac{1}{4}$ of a bushel of corn every day; what part of a bushel did he give every day?

7. What part of a bushel is $\frac{1}{4}$ of $\frac{1}{2}$ of a bushel?

8. At $\frac{3}{5}$ of a dollar per yard, what cost $\frac{1}{3}$ of a yard of merino? $\frac{1}{5}$ of a yard at $\frac{1}{3}$ of a dollar a yard?

9. What is the difference between $\frac{1}{4}$ of $\frac{1}{5}$ of a dollar, and $\frac{1}{5}$ of $\frac{1}{4}$ of a dollar?

10. A general sent $\frac{1}{6}$ of his army on an expedition, but $\frac{1}{4}$ of that detachment were slain in battle; what part of the whole army was thus lost?

11. What is $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{6}$?

12. An army had 2 battles; in the first it lost $\frac{1}{7}$ of its men, and in the second $\frac{1}{6}$ of the remainder; what part of the original army remained?

13. If a dime is $\frac{1}{10}$ of a dollar, what part of a dollar is a half-dime?

14. What is $\frac{1}{3}$ of $\frac{3}{8}$? $\frac{1}{4}$ of $\frac{1}{8}$? $\frac{1}{5}$ of $\frac{5}{8}$? $\frac{1}{7}$ of $\frac{7}{8}$?

15. A had $\frac{1}{4}$ of a ton of hay, and sold $\frac{1}{2}$ of it to B, who sold $\frac{1}{3}$ of his to C; what part of a ton had each then?

16. How is $\frac{2}{3}$ of a number found?

Ans. By dividing it by 3 to find $\frac{1}{3}$, and multiplying that by 2 to find $\frac{2}{3}$.

17. What are $\frac{2}{3}$ of $\frac{3}{4}$?

FIRST ANALYSIS.—One-third of $\frac{3}{4}$ is $\frac{1}{4}$; two-thirds of $\frac{3}{4}$ must be 2 times $\frac{1}{4}$, that is, $\frac{2}{4}$ or $\frac{1}{2}$. This result may be obtained by finding $\frac{2}{3}$ of the numerator, leaving the denominator unchanged.

SECOND ANALYSIS.— $\frac{1}{3}$ of $\frac{3}{4}$ is $\frac{1}{4}$; $\frac{2}{3}$ of $\frac{3}{4}$ must be 2 times $\frac{1}{4}$, that is $\frac{2}{4}$ or $\frac{1}{2}$. This result may be obtained by multiplying the numerators together for a new numerator, and the denominators together for a new denominator.

THIRD ANALYSIS.— $\frac{3}{4}$ is $\frac{9}{12}$; and $\frac{2}{3}$ of $\frac{9}{12}$ must be $\frac{6}{12}$ or $\frac{1}{2}$.

18. At $\frac{3}{8}$ of a dollar a bushel, what costs $\frac{1}{3}$ of a bushel of apples? $\frac{2}{3}$ of a bushel? $\frac{3}{4}$? $\frac{5}{8}$? $\frac{7}{8}$?

19. At $\frac{4}{5}$ of a dollar a yard, what costs $\frac{1}{4}$ of a yard of satinet? $\frac{3}{4}$ of a yard? $\frac{3}{8}$? $\frac{1}{2}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{2}{3}$?

20. What is $\frac{3}{5}$ of $\frac{6}{7}$? $\frac{2}{5}$ of $\frac{5}{12}$? $\frac{4}{5}$ of $\frac{5}{8}$? $\frac{4}{5}$ of $\frac{9}{10}$?

21. A man having $\frac{7}{10}$ of a dollar spent $\frac{6}{7}$ of what he had; what part of a dollar had he left?

22. Mary cut and took out $\frac{1}{6}$ of a cake, and then her mother cut the rest into 4 equal pieces; after 1 of these pieces had been taken, what part of the whole cake remained?

23. After $\frac{2}{7}$ of a pie had been taken, the rest was cut into 6 equal pieces; after 1 of these pieces had been taken, what part of the whole pie remained? Was the remainder more or less than half of the pie?

24. What are $\frac{2}{7}$ of $\frac{7}{9}$? $\frac{3}{4}$ of $\frac{8}{9}$? $\frac{4}{7}$ of $\frac{7}{10}$? $\frac{5}{7}$ of $\frac{2}{5}$?

25. Hubert's age is $\frac{5}{12}$ of his father's, and Lily's age is $\frac{7}{10}$ of Hubert's; what part of her father's age is Lily's age?

26. What are $\frac{9}{10}$ of $\frac{20}{7}$? $\frac{7}{10}$ of $\frac{20}{1}$? $\frac{3}{10}$ of $\frac{20}{9}$?

27. What is a fraction of a fraction called?

Ans. A *compound* fraction.

28. What is the process called by which the value of a compound fraction is found?

Ans. Multiplication of fractions.

29. How do you multiply one fraction by another?

30. What is $\frac{3}{7}$ of $\frac{5}{11}$? $\frac{2}{9}$ of $\frac{4}{7}$? $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{11}$ of $\frac{1}{11}$?

LESSON XV.

1. What is $\frac{1}{2}$ of 3?

FIRST ANALYSIS.—3 is 6 halves; $\frac{1}{2}$ of 6 halves is 3 halves, or $\frac{3}{2}$; therefore $\frac{1}{2}$ of 3 is $\frac{3}{2}$, or $1\frac{1}{2}$.

SECOND ANALYSIS.— $\frac{1}{2}$ of 1 is $\frac{1}{2}$; $\frac{1}{2}$ of 3 must be 3 times $\frac{1}{2}$ or $\frac{3}{2} = 1\frac{1}{2}$.

THIRD ANALYSIS.—3 is $2 + 1$. $\frac{1}{2}$ of 2 is 1, and $\frac{1}{2}$ of the remaining 1 is $\frac{1}{2}$; therefore $\frac{1}{2}$ of 3 is $1 + \frac{1}{2}$, or $1\frac{1}{2}$, which is equal to $\frac{3}{2}$. Therefore, $\frac{1}{2}$ of 3 is $\frac{3}{2}$.

NOTE.—The third method is *independently* applicable only to those cases in which the given number is separable into 1 and a number divisible by the denominator.

2. What is $\frac{1}{2}$ of 5? 7? 9? 11? 13? 15?

3. What is $\frac{1}{3}$ of 2? 4? 5? 7? 8? 10?

4. What is $\frac{1}{4}$ of 2? 3? 5? 6? 7? 9?

5. What is $\frac{1}{5}$ of 2? 3? 4? 6? 12? 13?

6. What is $\frac{1}{6}$ of 5? 7? 8? 9? 10? 17?

7. What is $\frac{1}{7}$ of 6? 8? 9? 17? 25? 33?

8. What is $\frac{1}{8}$ of 7? 9? 19? 28? 37? 47?

9. What is $\frac{1}{9}$ of 8? 10? 20? 31? 41? 52?

10. What is $\frac{1}{10}$ of 9? 11? 23? 34? 47? 59?

11. What is $\frac{1}{11}$ of 7? 8? 23? 35? 47? 59?

12. If 12 bushels of wheat cost 15 dollars, what is the cost of 1 bushel?

ANALYSIS.—If 12 bushels cost 15 dollars, 1 bushel costs $\frac{1}{12}$ of 15 dollars, or $\frac{15}{12}$ of a dollar, equal to $1\frac{3}{4}$ or $1\frac{1}{4}$ dollars; therefore if 12 bushels cost 15 dollars, 1 bushel costs $1\frac{1}{4}$ dollars.

13. If 3 oranges cost 13 cents, what is the average cost?

14. If 9 pounds of sugar are sold for a dollar, that is, 100 cents, what is the cost per pound?

15. If you walk 15 miles in 4 hours, what is the average per hour?

16. If 8 sheep are sold for 45 dollars, what is the average price?

17. If a quire of paper, (24 sheets,) is sold for 18 cents, what is the cost per sheet?

18. What do 6 hats average, if they cost 23 dollars?

19. What are $\frac{2}{3}$ of 4?

ANALYSIS.— $\frac{2}{3}$ of 4 are 2 times $\frac{1}{3}$ of 4; $\frac{1}{3}$ of 4 is $\frac{4}{3}$; 2 times $\frac{4}{3}$ are $\frac{8}{3}$, or $1\frac{2}{3}$; therefore $\frac{2}{3}$ of 4 are $1\frac{2}{3}$.

20. What are $\frac{2}{3}$ of 4? 5? 7? 8? 10? 11?

21. What are $\frac{3}{4}$ of 5? 6? 7? 9? 18? 31?

22. What are $\frac{4}{5}$ of 6? 7? 13? 24? 41? 52?

23. What are $\frac{5}{6}$ of 7? 14? 21? 40? 47? 49?

24. What are $\frac{5}{7}$ of 8? 16? 24? 32? 40? 48?

25. What are $\frac{3}{8}$ of 9? 18? 27? 36? 45? 55?

26. What are $\frac{7}{9}$ of 10? 20? 40? 50? 70? 80?

27. What are $\frac{7}{10}$ of 11? 33? 55? 77? 88? 99?

28. If 3 tons of coal cost 8 dollars, how much do 2 tons cost?

ANALYSIS.—If 3 tons cost 8 dollars, 1 ton costs $\frac{1}{3}$ of 8 dollars, or $\frac{8}{3}$ of a dollar, and 2 tons cost 2 times $\frac{8}{3}$, that is $1\frac{6}{3}$ of a dollar, or $5\frac{1}{3}$ dollars.

OR: If 3 tons cost 8 dollars, 1 ton costs $\frac{1}{3}$ of 8 dollars, or $2\frac{2}{3}$ dollars, and 2 tons cost 2 times $2\frac{2}{3}$ dollars, or $4\frac{4}{3}$, equal to $5\frac{1}{3}$ dollars.

29. If 4 yards of cloth cost 19 dollars, how much do 3 yards cost? 5 yards? 6 yards? 7 yards?

30. If 12 hats cost 33 dollars, what do 9 cost?

31. If 11 acres of wild land cost 36 dollars, what do 7 acres cost? 9 acres? 10 acres?

32. At the rate of 44 miles in 5 days, how far would you walk in 3 days? 4 days? 6 days?

33. At the rate of 8 pounds of soap for 38 cents, how much will 10 pounds cost? 11 pounds?

34. If a man saves at the rate of 64 dollars in 5 months, how much would he save in 12 months?

35. At the rate of 237 miles in 10 hours, how far does a train move in 7 hours? In 3 hours?

36. If the fare is at the rate of 29 cents for 10 miles, what is it for 25 miles? For 31 miles?

37. A man raised 11 acres of wheat for another, and was to have the crop of 5 acres for his labor; the whole crop was 225 bushels; what was the man's share?

38. Fifty-four is $\frac{5}{9}$ of what number?

39. Forty-five is $\frac{7}{8}$ of what number?

LESSON XVI.

1. At 7 dollars a yard, what cost $2\frac{1}{2}$ yards of cloth?

FIRST ANALYSIS.—At 7 dollars a yard, $2\frac{1}{2}$ yards cost $2\frac{1}{2}$ times 7 dollars. 2 times 7 dollars are 14 dollars, and $\frac{1}{2}$ of 7 dollars is $3\frac{1}{2}$ dollars; 14 dollars and $3\frac{1}{2}$ dollars are $17\frac{1}{2}$ dollars; therefore at 7 dollars a yard $2\frac{1}{2}$ yards of cloth cost $17\frac{1}{2}$ dollars.

SECOND ANALYSIS.—At 7 dollars a yard, 2 yards cost 2 times 7 dollars, or 14 dollars, and $\frac{1}{2}$ of a yard costs $\frac{1}{2}$ of 7 dollars, or $3\frac{1}{2}$ dollars; therefore $2\frac{1}{2}$ yards cost $14 + 3\frac{1}{2} = 17\frac{1}{2}$ dollars.

2. At 10 cents a quire, what cost $3\frac{1}{3}$ quires of paper? $5\frac{2}{3}$ quires? $3\frac{3}{4}$ quires?

3. At 75 cents a gallon, what cost $4\frac{1}{2}$ gallons of alcohol? $5\frac{1}{8}$ gallons?

4. At 40 dollars an acre, what cost $5\frac{1}{2}$ acres of land? $10\frac{2}{5}$ acres? $7\frac{5}{8}$ acres? $6\frac{2}{3}$ acres?

5. At 13 cents a pound, what cost $2\frac{1}{8}$ pounds of cheese? $3\frac{1}{4}$ pounds? $5\frac{1}{2}$ pounds?

6. At 5 dollars a week, how much would $10\frac{5}{7}$ weeks' board cost? $12\frac{3}{7}$ weeks' board? $20\frac{2}{7}$ weeks' board?

7. At 13 cents a yard, what cost $7\frac{4}{9}$ yards of ribbon? $5\frac{5}{9}$ yards? $6\frac{7}{9}$ yards? $3\frac{8}{9}$ yards?

8. If a thousand dollars earn 60 dollars in 12 months, how much does it earn in $2\frac{1}{10}$ months? $3\frac{3}{10}$ months? $4\frac{5}{10}$? $5\frac{6}{10}$? $6\frac{7}{10}$? $8\frac{9}{10}$? $10\frac{8}{10}$? $11\frac{4}{10}$?

9. How much are $9\frac{5}{11}$ times 16?

10. If a hundred dollars earn 5 dollars in a year, how much will it earn in $2\frac{5}{12}$ years? $6\frac{7}{12}$ years?

11. A woman sold at a store $12\frac{1}{2}$ pounds of rags at 5 cents a pound, and $2\frac{5}{8}$ dozen of eggs at 12 cents a dozen; she took for pay $4\frac{1}{4}$ yards of ribbon at 10 cents a yard, and $\frac{1}{4}$ of a pound of tea at 75 cents a pound; how much was still due her?

12. What is the sum of $3\frac{1}{4} \times 9 + 5\frac{1}{8} \times 9$?

13. What is $7 \times \frac{4}{5} - \frac{7}{10} \times 5$?

14. Multiply $2\frac{4}{9}$ times 9 by $1\frac{1}{5}$ times 10.

15. How many are $2\frac{5}{6}$ times 18?

16. How much is $\frac{2}{3}$ of $10\frac{1}{2}$?

FIRST ANALYSIS.— $10\frac{1}{2}$ is $\frac{21}{2}$; $\frac{1}{3}$ of $\frac{21}{2}$ is $\frac{7}{2}$, and $\frac{2}{3}$ of $\frac{21}{2}$ is 2 times $\frac{7}{2}$, or $\frac{14}{2}$, equal to 7. Therefore $\frac{2}{3}$ of $10\frac{1}{2}$ is 7.

SECOND ANALYSIS.— $\frac{1}{3}$ of 10 is $3\frac{1}{3}$, and $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$: $3\frac{1}{3} + \frac{1}{6} = 3\frac{2}{6} + \frac{1}{6} = 3\frac{3}{6}$, or $3\frac{1}{2}$: if $3\frac{1}{2}$ is $\frac{1}{3}$ of $10\frac{1}{2}$, 2 times $3\frac{1}{2}$, or 7, must be $\frac{2}{3}$ of $10\frac{1}{2}$.

17. At $7\frac{1}{2}$ dollars a barrel, what does $\frac{1}{2}$ of a barrel of flour cost? $\frac{1}{4}$ of a barrel? $\frac{3}{4}$ of a barrel?

18. At $18\frac{3}{4}$ cents a pound, how much does $\frac{5}{8}$ of a pound of butter cost? $\frac{7}{8}$ of a pound?

19. At $4\frac{5}{8}$ dollars per cord, how much does $\frac{3}{4}$ of a cord of wood cost? $\frac{5}{8}$ of a cord? $\frac{7}{8}$ of a cord?

20. At $9\frac{1}{2}$ pounds of sugar for a dollar, how much will $\frac{7}{10}$ of a dollar buy? $\frac{3}{10}$ of a dollar?

21. What are $\frac{5}{6}$ of $13\frac{2}{3}$? $\frac{1}{6}$ of $18\frac{6}{7}$? $\frac{2}{7}$ of $14\frac{7}{9}$?

22. At the rate of 120 dollars for 11 acres of land, what does $\frac{4}{5}$ of an acre cost? $\frac{9}{10}$ of an acre?

23. At $\frac{2}{5}$ of a dollar per bushel, what cost $4\frac{1}{4}$ bushels of wheat?

FIRST ANALYSIS.—At $\frac{2}{5}$ of a dollar per bushel, 4 bushels will cost 4 times $\frac{2}{5}$, or $\frac{8}{5}$ of a dollar, and $\frac{1}{4}$ of a bushel will cost $\frac{1}{4}$ of $\frac{2}{5}$ of a dollar, or $\frac{2}{20}$ of a dollar; $4\frac{1}{4}$ bushels will cost $\frac{8}{5} + \frac{2}{20}$, or $\frac{16}{10} + \frac{1}{10} = \frac{17}{10}$ of a dollar, equal to $2\frac{1}{10}$ dollars.

SECOND ANALYSIS.—At $\frac{2}{5}$ of a dollar per bushel, $4\frac{1}{4}$ bushels, or $\frac{17}{4}$ of a bushel, will cost $\frac{17}{4}$ of $\frac{2}{5} = \frac{17}{10}$ of a dollar, or $2\frac{1}{10}$ dollars.

24. At $\frac{2}{3}$ of a dollar a yard, what cost $5\frac{1}{2}$ yards of cloth? $6\frac{1}{4}$ yards? $8\frac{5}{8}$ yards?

25. At $\frac{3}{4}$ of a dollar a day, how much does a man earn in $2\frac{1}{2}$ days? $5\frac{1}{2}$ days? $10\frac{1}{2}$ days?

26. At $\frac{7}{10}$ of a dollar a pound, what cost $3\frac{5}{8}$ pounds of tea? $4\frac{3}{4}$ pounds? $12\frac{1}{2}$ pounds?

27. At $\frac{3}{16}$ of a dollar a pound, what cost $2\frac{1}{2}$ pounds of coffee? $3\frac{1}{4}$ pounds? $6\frac{3}{8}$ pounds?

28. At $\frac{7}{8}$ of a dollar a gallon, what cost $3\frac{1}{2}$ gallons of oil? $6\frac{1}{4}$ gallons? $2\frac{3}{4}$ gallons? $1\frac{7}{8}$ gallons?

29. What cost $8\frac{1}{2}$ quires of paper at $\frac{3}{20}$ of a dollar per quire? $10\frac{1}{3}$ quires? $12\frac{1}{4}$ quires?

30. At $\frac{9}{10}$ of a mile an hour, how far does a stream run in $8\frac{1}{3}$ hours? $9\frac{3}{10}$ hours? $12\frac{3}{4}$ hours?

LESSON XVII.

1. What cost $3\frac{1}{2}$ yards of silk at $2\frac{1}{4}$ dollars per yard?

NOTE.—This question can be analyzed either by reducing the mixed numbers to fractions, and then proceeding as usual in multiplying one fraction by another, or by the following method of

ANALYSIS.—At $2\frac{1}{4}$ dollars per yard, 3 yards cost 3 times $2\frac{1}{4}$, or $6\frac{3}{4}$ dollars, and $\frac{1}{2}$ of a yard costs $\frac{1}{2}$ of $2\frac{1}{4}$ dollars, or $1\frac{1}{8}$ dollars; therefore $3\frac{1}{2}$ yards cost $6\frac{3}{4} + 1\frac{1}{8} = 6\frac{6}{8} + 1\frac{1}{8} = 7\frac{7}{8}$ dollars.

2. What cost $4\frac{1}{2}$ pounds of coffee at $12\frac{1}{2}$ cents a pound?

3. What cost $5\frac{1}{4}$ gallons of wine at $2\frac{1}{4}$ dollars a gallon?

4. What cost $2\frac{1}{3}$ yards of broadcloth at $5\frac{1}{2}$ dollars per yard?

5. At the rate of $8\frac{3}{5}$ miles an hour, how far would a vessel sail in $4\frac{3}{4}$ hours?

6. Bought a violin for $10\frac{3}{5}$ dollars, and sold it for $2\frac{5}{6}$ times what it cost; how much did I get for it?

7. Merwin is $9\frac{5}{8}$ years old, and his father is $3\frac{1}{2}$ times as old; how old is his father?

8. What cost $4\frac{3}{7}$ weeks' board at $3\frac{3}{4}$ dollars a week?

9. What cost $2\frac{5}{8}$ pecks of clover-seed at $1\frac{2}{5}$ dollars a peck?

10. Multiply $10\frac{1}{9}$ by $9\frac{1}{10}$: $4\frac{1}{4}$ by $4\frac{1}{4}$.

11. Multiply $7\frac{1}{5}$ by $5\frac{1}{7}$: $5\frac{1}{5}$ by $5\frac{1}{5}$.

12. Multiply $9\frac{1}{2}$ by $9\frac{1}{2}$: $6\frac{1}{6}$ by $6\frac{1}{6}$.

13. Multiply $7\frac{2}{9}$ by $7\frac{2}{9}$: $8\frac{3}{10}$ by $8\frac{3}{10}$.

14. A man borrowed a hundred dollars of another, and paid for its use $7\frac{1}{2}$ dollars every year; how much would the payments amount to in $6\frac{7}{12}$ years?

15. How many are $8\frac{2}{3}$ times $10\frac{2}{3}$?

16. What cost $6\frac{1}{2}$ yards of muslin at $16\frac{2}{3}$ cents per yard?

17. What cost $5\frac{3}{8}$ cords of wood at $5\frac{1}{4}$ dollars per cord?

18. At $\frac{2}{5}$ of a dollar apiece, how many decanters can be bought for 4 dollars?

ANALYSIS.—At $\frac{2}{5}$ of a dollar apiece, 4 dollars will buy as many decanters as $\frac{2}{5}$ are contained times in 4. Since 4 is $\frac{2}{5}$, and $\frac{2}{5}$ are contained in $\frac{2}{5}$ 10 times, 10 decanters can be bought for 4 dollars.

19. At $\frac{3}{4}$ of a dollar a day, in how many days would a person earn 6 dollars?

20. At $\frac{4}{5}$ of a dollar a day, in how many days would a person's board amount to 24 dollars?

21. At $\frac{2}{3}$ of a mile a minute, how long would a train be in moving 40 miles?

22. At the rate of 6 apples for 5 cents, how many apples will $12\frac{1}{2}$ cents buy?

23. At $\frac{5}{8}$ of a dollar a pair, how many pairs of gloves can be bought for 15 dollars?

24. How many times $\frac{5}{7}$ are in 12? 20? 25?

25. If I pay 10 dollars for $\frac{5}{9}$ of a ton of hay, at what rate per ton is that?

26. If $\frac{8}{9}$ of a yard of brocade sell for 4 dollars, at what rate is that per yard?

27. How many times $\frac{9}{10}$ are in 10? 15? 90?

28. Traded 16 bushels of oats at $\frac{5}{16}$ of a dollar a bushel, for apples at $\frac{3}{8}$ of a dollar a bushel; how many bushels of apples should I receive?

29. How many times $\frac{8}{11}$ are $\frac{2}{3}$ of 12?

30. How many times $\frac{7}{12}$ are $\frac{1}{3}$ of 21?

31. If a soldier's ration of meat is $\frac{9}{16}$ of a pound a day, how long would 9 pounds last him?

32. How many times $\frac{9}{20}$ are $\frac{3}{5}$ of 15?

33. How many times $\frac{12}{17}$ are $\frac{4}{9}$ of 27?

34. Gave 50 dollars for $\frac{2}{5}$ of an acre of land; at what rate per acre was it?

35. Gave 2 dimes for $\frac{4}{5}$ of a quire of paper; at what rate per quire was that?

36. Gave 2 dollars for $\frac{5}{6}$ of a dozen of brooms; what was the rate per dozen?

37. How many times $\frac{8}{15}$ are $\frac{4}{5}$ of 20?

38. How many times $\frac{5}{18}$ are $\frac{5}{9}$ of 45?

LESSON XVIII.

1. How many times $\frac{3}{8}$ are there in $\frac{15}{16}$?

FIRST ANALYSIS.— $\frac{3}{8}$ equal $\frac{6}{16}$: 6-sixteenths are contained in 15-sixteenths $2\frac{1}{2}$ times; therefore there are $2\frac{1}{2}$ times $\frac{3}{8}$ in $\frac{15}{16}$.

SECOND ANALYSIS.—3 is contained in $\frac{15}{8}$ $\frac{5}{16}$ times, and $\frac{1}{8}$ of 3, that is $\frac{3}{8}$, is contained 8 times $\frac{5}{16}$, which is $\frac{5}{2}$, or $2\frac{1}{2}$ times.

2. How many times $\frac{2}{5}$ are in $\frac{9}{10}$?

3. How many times $\frac{1}{3}$ are in $\frac{11}{2}$?

4. How many times $\frac{2}{3}$ are in $\frac{22}{3}$?

5. How many times $\frac{1}{4}$ are in $\frac{13}{3}$?

6. How many times $\frac{3}{4}$ are in $\frac{5}{6}$? In $\frac{7}{8}$? In $\frac{9}{10}$?

7. How many times $\frac{1}{5}$ are in $\frac{5}{6}$? How many times $\frac{2}{3}$?

8. How many times $\frac{3}{5}$ are in $\frac{18}{9}$? In $\frac{24}{5}$? In $\frac{27}{8}$?

9. How many times $\frac{1}{4}$ are in $\frac{1}{3}$? In $\frac{1}{2}$?

10. How many times $\frac{1}{5}$ are in $\frac{1}{4}$? In $\frac{1}{3}$? In $\frac{1}{2}$?

11. How many times $\frac{1}{6}$ are in $\frac{1}{5}$? In $\frac{1}{4}$? In $\frac{1}{3}$?

12. How many times $\frac{1}{7}$ are in $\frac{1}{6}$? In $\frac{1}{5}$? In $\frac{1}{4}$?

13. From these analyses, form a rule for dividing one fraction by another.

14. At $\frac{2}{5}$ of a dollar a bushel, how many bushels of apples can be bought for $\frac{1}{2}$ of a dollar?

15. At $\frac{3}{10}$ of a dollar a pound, how many pounds of confectionery can be bought for $\frac{1}{6}$ of a dollar?

16. At $\frac{3}{16}$ of a dollar a pound, how many pounds of honey can be bought for $1\frac{1}{2}$ dollars?

17. At $\frac{5}{16}$ of a dollar a yard, how many yards of chintz can be bought for $12\frac{1}{2}$ dollars?

18. At $\frac{5}{8}$ of a dollar a pint, how many pints of wine can be bought for a dollar? For $\frac{1}{2}$ of a dollar?

19. How many wine-glasses, each holding $\frac{1}{8}$ of a quart, can be filled from a bottle containing $\frac{3}{4}$ of a quart?

20. How many bottles, each containing $\frac{7}{32}$ of a gallon, can be filled from a cask containing $31\frac{1}{2}$ gallons?

21. Bought $2\frac{1}{4}$ yards of cloth for $7\frac{1}{2}$ dollars; what was the price per yard?

22. Divide $5\frac{5}{12}$ by $1\frac{1}{9}$. By $1\frac{2}{3}$. By $1\frac{3}{10}$. By $9\frac{2}{7}$.

23. If $\frac{3}{4}$ of a yard of silk cost $\frac{9}{10}$ of a dollar, what is the price per yard?

FIRST ANALYSIS.—If $\frac{3}{4}$ of a yard cost $\frac{9}{10}$ of a dollar, 1 yard will cost as many dollars as $\frac{3}{4}$ is contained times in $\frac{9}{10}$; 3 is contained $\frac{3}{10}$ times, and $\frac{1}{4}$ of 3, or $\frac{3}{4}$ is contained 4 times $\frac{3}{10}$, or $\frac{12}{10}$, or $1\frac{2}{10}$, or $1\frac{1}{5}$ times. Therefore 1 yard would cost $1\frac{1}{5}$ dollars.

SECOND ANALYSIS.—If $\frac{3}{4}$ of a yard cost $\frac{9}{10}$ of a dollar, $\frac{1}{4}$ of a yard will cost $\frac{1}{3}$ of $\frac{9}{10}$, or $\frac{3}{10}$, of a dollar; and $\frac{4}{4}$ or 1 yard will cost 4 times $\frac{3}{10}$, or $\frac{12}{10}$, or $1\frac{2}{10}$, or $1\frac{1}{5}$ dollars.

NOTE.—Those to whom the truth of the reasoning in the first analysis is not apparent, can use the second method.

24. If $\frac{1}{2}$ of a yard of silk cost $\frac{5}{8}$ of a dollar, what is the price per yard?

25. If $\frac{2}{3}$ of a yard of linen cost $\frac{3}{4}$ of a dollar, what is the price per yard?

26. If $\frac{3}{4}$ of a bushel of wheat cost $\frac{3}{5}$ of a dollar, what is the price per bushel?

27. If a person walks $\frac{3}{5}$ of a mile in $\frac{1}{6}$ of an hour, at what rate per hour is that?

28. If a locomotive moves $\frac{4}{5}$ of a mile in $\frac{9}{10}$ of a minute, at what rate per minute does it move?

29. $\frac{2}{3}$ are $\frac{3}{4}$ of what number? $\frac{2}{3}$ are $\frac{3}{5}$ of what?

30. $\frac{2}{3}$ are $\frac{2}{5}$ of what number? $\frac{2}{3}$ are $\frac{4}{5}$ of what?

31. $\frac{3}{4}$ are $\frac{3}{5}$ of what number? $\frac{3}{4}$ are $\frac{4}{5}$ of what?

32. $\frac{2}{5}$ are $\frac{5}{6}$ of what number? $\frac{3}{5}$ are $\frac{5}{6}$ of what?

33. $\frac{7}{8}$ are $\frac{5}{9}$ of what number? $\frac{5}{9}$ are $\frac{7}{8}$ of what?

34. $\frac{8}{9}$ are $\frac{2}{7}$ of what number?

35. James has $\frac{5}{8}$ of a dollar, which is $\frac{5}{6}$ of the money which John has; how much has John?

36. Mary, having spent some money, has $\frac{3}{5}$ of a dime left, which is $\frac{4}{15}$ of what she had at first; how much had she at first?

37. How many times are $\frac{3}{10}$ contained in $\frac{9}{10}$?

38. How many times are $\frac{7}{9}$ contained in $\frac{9}{9}$?

39. By selling gloves at $\frac{6}{20}$ of a dollar a pair, a man received $\frac{12}{5}$ of a dollar; how many pairs did he sell?

40. If $\frac{1}{5}$ of a dollar are distributed in prizes of $\frac{2}{5}$ of a dollar each, how many prizes are there?

LESSON XIX.

1. At $3\frac{1}{2}$ dollars apiece, how many hats can be purchased with $31\frac{1}{2}$ dollars?

NOTE.—After reducing mixed numbers to improper fractions, proceed by analyses given before.

2. At $2\frac{3}{4}$ dollars each, how many vests can be purchased with $30\frac{1}{4}$ dollars?

3. At $1\frac{3}{8}$ dollars a bushel, how many bushels of wheat can be bought with $20\frac{5}{8}$ dollars?

4. If $6\frac{2}{3}$ yards of cloth cost $21\frac{2}{3}$ dollars, what is the price per yard?

5. How many times $1\frac{1}{2}$ are $3\frac{1}{2}$? $4\frac{3}{4}$?

6. How many times $2\frac{2}{5}$ are $8\frac{3}{5}$? $6\frac{3}{10}$?

7. Spent $10\frac{1}{8}$ dollars in buying molasses at $\frac{9}{16}$ of a dollar a gallon; how many gallons did I buy?

8. If a person wastes $\frac{1}{10}$ of a dollar a day, in how many days would he waste $7\frac{1}{2}$ dollars?

9. If a person saves $9\frac{7}{10}$ dollars per month, in how many months would he save $116\frac{2}{5}$ dollars?

10. How many times $3\frac{5}{6}$ are $10\frac{4}{5}$? $12\frac{2}{3}$? $11\frac{1}{2}$?

11. How many times $2\frac{1}{4}$ are 9? $11\frac{1}{4}$? $13\frac{1}{2}$?

12. How many times $3\frac{1}{2}$ are 7? $10\frac{1}{2}$? 14?

13. How many times $4\frac{3}{4}$ are $9\frac{1}{2}$? $11\frac{1}{4}$? $23\frac{3}{4}$?

14. How many times $5\frac{1}{3}$ are 16? $26\frac{2}{3}$? $37\frac{1}{3}$?

15. How many times $6\frac{2}{3}$ are $13\frac{1}{3}$? 40? $53\frac{1}{3}$?

16. How many times $7\frac{1}{5}$ are 36? $10\frac{4}{5}$? $14\frac{2}{5}$?

17. How many times $8\frac{2}{5}$ are $25\frac{1}{5}$? $12\frac{3}{5}$? $10\frac{1}{2}$?

18. How many times $4\frac{1}{6}$ are 5? 25? $6\frac{1}{4}$?

19. How many times $3\frac{5}{6}$ are $5\frac{1}{9}$? $5\frac{3}{4}$?

20. How many times $1\frac{1}{7}$ are $2\frac{3}{7}$? $3\frac{3}{7}$? $4\frac{4}{7}$? $5\frac{5}{7}$?

21. How many times $2\frac{1}{8}$ are $4\frac{1}{4}$? $6\frac{3}{8}$? $8\frac{1}{2}$? $10\frac{5}{8}$? $12\frac{3}{4}$?
 $14\frac{7}{8}$? 17? $19\frac{1}{8}$? $21\frac{1}{4}$?

22. How many times $1\frac{1}{9}$ are 10? $3\frac{1}{3}$? $6\frac{2}{3}$?

23. How many times $2\frac{1}{10}$ are 21? $10\frac{1}{2}$? $5\frac{1}{4}$?

24. How many times $1\frac{4}{11}$ are $7\frac{1}{2}$? $3\frac{3}{4}$? $1\frac{7}{8}$?

25. How many times $2\frac{5}{12}$ are $14\frac{1}{2}$? $7\frac{1}{4}$? $3\frac{5}{8}$?

LESSON XX.

1. At 18 cents a pound, what cost $\frac{3}{4}$ of a pound of raisins? $\frac{5}{8}$ of a pound? $\frac{7}{8}$ of a pound?

2. A farmer having 120 sheep, exchanged $\frac{3}{10}$ of them for 5 sheep of a better kind; how many had he then?

3. He afterwards gave his neighbor 2 of his best for 10 common sheep; how many had he then?

4. How many dollars in $\frac{6}{10}$ of a dollar?

5. Make a brief rule for finding the value of an improper fraction.

6. John, Joseph, Harriet and Susan spent a day in picking berries; John and Harriet picked into one basket $\frac{1}{2}$ of a bushel; Joseph picked $\frac{3}{8}$ of a bushel in his basket, and Susan $\frac{5}{16}$ of a bushel in hers; what was the average picking?

7. After spending $\frac{2}{5}$ of my money, I found that I had money enough to pay for 9 weeks' board at 4 dollars a week; how much money had I at first?

8. After cutting $7\frac{1}{2}$ yards from a piece of cloth, a tailor made from the rest 9 coats, averaging $3\frac{3}{4}$ yards each; how much cloth was in the piece at first?

9. A smith had $185\frac{1}{2}$ pounds of iron, from which he took $145\frac{1}{2}$ pounds for a wagon, and made the rest into 35 horse-shoes; what weight did the shoes average?

10. A piece of land, containing $46\frac{7}{8}$ acres, fell to 3 heirs to be divided equally, but, before division, one of them sold his share to another; how many acres did the latter receive?

11. What is $\frac{5}{7}$ of $\frac{6}{7}$?

12. What *part* of 6 is 5? What part of $\frac{6}{7}$ is $\frac{5}{7}$?

13. What is $\frac{3}{10}$ of $\frac{7}{10}$?

14. What part of 7 is 3? What part of $\frac{7}{10}$ is $\frac{3}{10}$?

15. What is $\frac{2}{3}$ of $\frac{3}{4}$? What part of $\frac{3}{4}$ is $\frac{2}{3}$?

16. What part of 1 is $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{5}$ of $\frac{1}{2}$?

17. What part of 1 is $\frac{2}{5}$ of $\frac{3}{7}$? $\frac{3}{7}$ of $\frac{2}{5}$?

18. What part of 2 is $\frac{1}{3}$ of $\frac{1}{4}$?

SUGGESTION.— $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{1}{4}$ of 1, which is $\frac{1}{12}$ of $\frac{1}{2}$ of 2, or $\frac{1}{12}$ of 2.

19. What part of 2 are $\frac{2}{3}$ of $\frac{3}{4}$? What part of 3?
20. What part of 3 are $\frac{4}{5}$ of $\frac{5}{6}$? What part of 4?
21. What is $\frac{3}{5}$ of $\frac{5}{9}$? What part of $\frac{3}{5}$ is $\frac{5}{9}$?
22. What part of 3 is $\frac{3}{5}$ of $\frac{5}{9}$? What part of 4?
23. What is $\frac{6}{7}$ of $\frac{7}{9}$? What part of $\frac{6}{7}$ is $\frac{7}{9}$?
24. What part of 2 is $\frac{6}{7}$ of $\frac{7}{9}$? What part of 5?
25. What part of 12 is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of 36?
26. What part of 8 is $\frac{2}{5}$ of $\frac{5}{6}$ of $\frac{3}{4}$ of 49?
27. What part of 20 is $\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{7}{12}$ of 72?
28. What part of 24 is $\frac{2}{9}$ of $\frac{4}{5}$ of $\frac{9}{10}$ of 100?
29. What part of 15 is $\frac{1}{8}$ of $\frac{7}{8}$ of $\frac{4}{5}$ of 120?
30. If you walk $15\frac{3}{4}$ miles in 4 hours, how many miles do you average per hour?
31. How many kegs of 5 gallons each, can be filled from a barrel containing $31\frac{1}{2}$ gallons of cider?
32. How many times could a $2\frac{1}{2}$ gallon can be filled from a cask of oil containing $37\frac{1}{2}$ gallons?
33. Bought 6 yards of cloth for $17\frac{1}{4}$ dollars; what was the price per yard? What cost 5 yards?
34. Bought 8 pounds of coffee for $1\frac{1}{3}$ dollars; what was the price per pound? What cost 7 pounds?
35. Bought 12 pounds of tea for $10\frac{1}{2}$ dollars; what was the price per pound? What cost 10 pounds?
36. Bought 10 pounds of raisins for $1\frac{7}{8}$ dollars; what was the price per pound? What cost 3 pounds?
37. If 3 pounds of figs cost $\frac{9}{20}$ of a dollar, what do 2 pounds cost? 5 pounds? 7 pounds?
38. Bought 9 yards of silk, at the rate of 2 yards for $3\frac{1}{4}$ dollars, and offered 3 five-dollar bills; how much change was due to me?
39. A lady bought 12 yards of satin, at the rate of 3 yards for $5\frac{3}{4}$ dollars, but having only 2 ten-dollar bills, the rest was charged; how much was charged?
40. If 14 pounds of butter cost $2\frac{1}{10}$ dollars, what cost $2\frac{1}{2}$ pounds? $1\frac{1}{4}$ pounds?
41. If 2 pounds of figs cost $\frac{3}{10}$ of a dollar, how many pounds can be bought for $1\frac{1}{4}$ dollars?

42. Sold a buggy for 54 dollars, which was $\frac{3}{4}$ of what it cost; what did it cost?

43. Three men agreed, if necessary, to make up equally between them $52\frac{1}{2}$ dollars to complete a subscription, but they only had to pay 40 dollars; how much less than his first subscription had each to pay?

44. If 9 pounds of sugar cost $76\frac{1}{2}$ cents, how much does $\frac{3}{4}$ of a pound cost? $2\frac{1}{2}$ pounds?

45. If $2\frac{1}{3}$ yards of muslin cost $14\frac{7}{12}$ cents, how much does $\frac{1}{2}$ of a yard cost? What do 5 yards cost?

46. If 9 bushels of apples cost $4\frac{1}{2}$ dollars, how many bushels would cost $10\frac{1}{2}$ dollars?

47. If $\frac{3}{4}$ of a peck of beans cost $\frac{1}{5}$ of a dollar, how many pecks would cost 4 dollars?

48. At $1\frac{2}{3}$ dollars a day, in how many days would my expenses be $18\frac{1}{3}$ dollars?

49. A man bought $6\frac{2}{3}$ yards of calico at $7\frac{1}{2}$ cents a yard, and gave in return eggs at 10 cents a dozen; how many dozen of eggs paid for the calico?

50. At 8 pounds of tea for $4\frac{4}{5}$ dollars, how much will cost $1\frac{1}{20}$ dollars? $\frac{3}{10}$ of a dollar?

51. Being asked his age, a man said that $\frac{6}{13}$ of it had passed since he was 21 years old; how old was he?

52. A boy said that $\frac{2}{5}$ of his age, added to his age, would make him 21; how old was he?

53. If $\frac{2}{3}$ of a yard of cloth cost $\frac{4}{5}$ of a dollar, what is the cost of $\frac{5}{8}$ of a yard? $\frac{7}{8}$ of a yard?

54. If, by selling land at $2\frac{5}{6}$ times what I gave for it, I gain 242 dollars, what did I give for it?

55. If by selling a watch at $\frac{11}{16}$ of what it cost, I lose 16 dollars, what did the watch cost me?

56. A man spent $\frac{3}{7}$ of his money, and then gained 640 dollars, when he found that he had $\frac{8}{7}$ of what he had at first; how much had he at first?

57. In an orchard $\frac{1}{3}$ of the trees are apple-trees, $\frac{1}{6}$ peach-trees, $\frac{1}{6}$ pear-trees, and 27 plum-trees; how many trees are in the orchard, and how many of each?

58. What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $2\frac{1}{2}$ times $2\frac{1}{2}$?

LESSON XXI.

1. $4\frac{4}{5}$ is $\frac{3}{8}$ of what number?

ANALYSIS.— $4\frac{4}{5}$ is $\frac{24}{5}$. If $\frac{24}{5}$ is $\frac{3}{8}$ of a number, $\frac{1}{8}$ of that number must be $\frac{1}{3}$ of $\frac{24}{5}$, or $\frac{8}{5}$, and the number must be 8 times $\frac{8}{5}$, or $6\frac{4}{5}$, or $12\frac{4}{5}$; therefore $4\frac{4}{5}$ is $\frac{3}{8}$ of $12\frac{4}{5}$.

2. $3\frac{3}{4}$ is $\frac{3}{5}$ of what number? $\frac{5}{6}$ of what? $\frac{3}{7}$?

3. $2\frac{2}{5}$ is $\frac{4}{9}$ of what number? $\frac{3}{4}$ of what? $\frac{6}{7}$?

4. $10\frac{2}{3}$ is $\frac{1}{2}$ of what number? $\frac{4}{5}$ of what? $\frac{8}{9}$?

5. $5\frac{5}{6}$ is $\frac{5}{6}$ of what number? $\frac{5}{7}$ of what? $\frac{7}{8}$?

6. $6\frac{3}{7}$ is $\frac{5}{12}$ of what number? $\frac{15}{16}$ of what? $\frac{3}{11}$?

7. $7\frac{7}{8}$ is $\frac{7}{9}$ of what number? $\frac{9}{10}$ of what? $\frac{7}{12}$?

8. 36 is $\frac{9}{10}$ of what number?

9. 3 is $\frac{4}{5}$ of what number? $\frac{2}{3}$ of what? $\frac{5}{9}$?

10. 5 is $\frac{3}{7}$ of what number?

11. 6 is $\frac{5}{8}$ of what number? $\frac{2}{11}$ of what? $\frac{4}{9}$?

12. 7 is $\frac{2}{3}$ of what number?

13. 11 is $\frac{3}{8}$ of what number? $\frac{5}{11}$ of what? $\frac{7}{12}$?

14. 17 is $\frac{5}{6}$ of what number?

15. 19 is $\frac{2}{3}$ of what number? $\frac{3}{4}$ of what? $\frac{7}{9}$?

16. 30 is $\frac{5}{7}$ of how many times 3?

ANALYSIS.—If 30 is $\frac{5}{7}$ of a number, $\frac{1}{5}$ of 30, or 6, is $\frac{1}{7}$ of that number, and 7 times 6, or 42, must be the number; 42 is 14 times 3; therefore 30 is $\frac{5}{7}$ of 14 times 3.

17. 28 is $\frac{4}{9}$ of how many times 9?

18. 25 is $\frac{5}{8}$ of how many times 10?

19. 32 is $\frac{8}{9}$ of how many times 12?

20. 18 is $\frac{2}{5}$ of how many times 5?

21. 20 is $\frac{5}{12}$ of how many times 3?

22. 21 is $\frac{3}{8}$ of how many times 4?

23. 64 is $\frac{4}{5}$ of how many times 8?

24. 72 is $\frac{8}{11}$ of how many times 3?

25. $12\frac{1}{2}$ is $\frac{5}{6}$ of how many times 3?

26. $18\frac{3}{4}$ is $\frac{3}{8}$ of how many times 20?

27. $31\frac{1}{4}$ is $\frac{5}{12}$ of how many times 15?

28. $37\frac{1}{2}$ is $\frac{3}{4}$ of how many times 10?

29. $5\frac{5}{7}$ is $\frac{8}{9}$ of how many times $\frac{5}{7}$?

30. 18 is $\frac{3}{7}$ of how many times $\frac{1}{5}$ of 30?
 31. 12 is $\frac{4}{9}$ of how many times $\frac{1}{3}$ of 15?
 32. 50 is $\frac{5}{7}$ of how many tenths of 100?
 33. 60 is $\frac{3}{5}$ of how many elevenths of 110?
 34. 56 is $\frac{7}{9}$ of how many twelfths of 96?
 35. 84 is $\frac{7}{8}$ of how many thirds of 48?
 36. $\frac{2}{3}$ of 27 is how many eighths of 48?

ANALYSIS.— $\frac{2}{3}$ of 27 is 18, and $\frac{1}{8}$ of 48 is 6; since 18 contains 6 three times, $\frac{2}{3}$ of 27 is $\frac{3}{8}$ of 48.

37. $\frac{1}{22}$ of 20 is how many ninths of 45?
 38. $\frac{3}{4}$ of 28 is how many fifths of 35?
 39. $\frac{2}{5}$ of 55 is how many sevenths of 14?
 40. $\frac{3}{5}$ of 40 is how many fifths of 30?
 41. $\frac{4}{5}$ of 60 is how many eighths of 64?
 42. $\frac{5}{6}$ of 84 is how many tenths of 140?
 43. $\frac{5}{7}$ of 63 is how many fourths of 60?
 44. $\frac{5}{8}$ of 72 is how many fourths of 60?
 45. $\frac{5}{9}$ of 81 is how many sixths of 90?
 46. $\frac{7}{10}$ of 120 is how many twelfths of 144?
 47. A farmer spent in town $6\frac{3}{5}$ dollars, which was $\frac{3}{5}$ of what he got for some hay; how much did he receive for the hay?

48. A farmer's wife bought at the store $9\frac{3}{4}$ dollars' worth of goods, which was $\frac{3}{4}$ of what was credited to her for butter and eggs; how much was still due her?

49. A man sold a horse for 75 dollars, which was $\frac{5}{8}$ of what he asked at first; how much did he take off?

50. How many persons, each worth $\frac{1}{10}$ of 50 thousand dollars, would be worth $\frac{7}{10}$ as much as a person worth 100 thousand dollars?

51. A man, being asked his age, said, "As many ninths of 45 as 15 is $\frac{3}{8}$ of, so many times 6 is my age." How old was he?

52. A said to B, "I raised 90 bushels of wheat; how many did you raise?" B replied, "I raised as many times 10 bushels as $\frac{2}{3}$ of your crop is fifths of 20." How many bushels did B raise?

53. $\frac{8}{15}$ of 105 is how many sevenths of 98?

LESSON XXII.

1. $\frac{4}{5}$ of 20 is $\frac{2}{3}$ of what number?

ANALYSIS.— $\frac{4}{5}$ of 20 is 16; if 16 is $\frac{2}{3}$ of a number, $\frac{1}{2}$ of 16, which is 8, is $\frac{1}{3}$ of that number; and 3 times 8, which is 24, must be that number. Therefore, $\frac{4}{5}$ of 20 is $\frac{2}{3}$ of 24.

2. $\frac{2}{3}$ of 18 is $\frac{3}{4}$ of what number? $\frac{3}{5}$ of what number?
3. $\frac{3}{4}$ of 20 is $\frac{3}{5}$ of what number? $\frac{3}{7}$ of what number?
4. $\frac{2}{5}$ of 60 is $\frac{3}{7}$ of what number? $\frac{3}{8}$ of what number?
5. $\frac{3}{5}$ of 75 is $\frac{5}{6}$ of what number? $\frac{9}{11}$ of what number?
6. $\frac{4}{5}$ of 40 is $\frac{4}{7}$ of what number? $\frac{8}{9}$ of what number?
7. $\frac{5}{6}$ of 42 is $\frac{7}{12}$ of what number? $\frac{5}{11}$ of what number?
8. $\frac{6}{7}$ of 56 is $\frac{3}{5}$ of what number? $\frac{4}{5}$ of what number?
9. $\frac{5}{7}$ of 84 is $\frac{5}{6}$ of what number? $\frac{12}{13}$ of what number?
10. $\frac{5}{8}$ of 80 is $\frac{2}{3}$ of what number? $\frac{5}{7}$ of what number?
11. $\frac{7}{9}$ of 72 is $\frac{4}{5}$ of what number? $\frac{7}{12}$ of what number?
12. $\frac{7}{10}$ of 100 is $\frac{5}{9}$ of what number? $\frac{10}{11}$ of what number?
13. $\frac{5}{11}$ of 132 is $\frac{4}{5}$ of what number? $\frac{5}{12}$ of what number?
14. $\frac{7}{12}$ of 168 is $\frac{2}{3}$ of what number? $\frac{7}{8}$ of what number?
15. $\frac{3}{4}$ of 32 is $\frac{2}{3}$ of how many times 9?

ANALYSIS.— $\frac{3}{4}$ of 32 is 24; if 24 is $\frac{2}{3}$ of a number, $\frac{1}{2}$ of 24, which is 12, is $\frac{1}{3}$ of that number, and 3 times 12, which is 36, must be that number. Since 36 is 4 times 9, $\frac{3}{4}$ of 32 is $\frac{2}{3}$ of 4 times 9

16. $\frac{2}{3}$ of 60 is $\frac{4}{7}$ of how many times 5?
17. $\frac{2}{5}$ of 90 is $\frac{3}{4}$ of how many times 6?
18. $\frac{3}{5}$ of 75 is $\frac{5}{6}$ of how many times 3?
19. $\frac{4}{5}$ of 100 is $\frac{8}{9}$ of how many times 10?
20. $\frac{5}{6}$ of 84 is $\frac{14}{5}$ of how many times $12\frac{1}{2}$?
21. $\frac{6}{7}$ of 77 is $\frac{3}{5}$ of how many times 11?
22. $\frac{5}{8}$ of 88 is $\frac{11}{10}$ of how many times $16\frac{2}{3}$?
23. $\frac{4}{9}$ of 36 is $\frac{1}{2}$ of how many times 14?
24. $\frac{7}{9}$ of 144 is $\frac{4}{5}$ of how many times 4?
25. $\frac{3}{10}$ of 100 is $\frac{5}{12}$ of how many times 8?
26. $\frac{7}{10}$ of 200 is $\frac{7}{6}$ of how many times 30?
27. $\frac{6}{11}$ of 121 is $\frac{2}{3}$ of how many times 9?
28. $\frac{5}{12}$ of 48 is $\frac{1}{4}$ of how many times 16?
29. $\frac{11}{12}$ of 180 is $\frac{5}{4}$ of how many times 6?

30. $\frac{4}{5}$ of 25 is $\frac{2}{3}$ of 6 times what number?

ANALYSIS.— $\frac{4}{5}$ of 25 is 20; if 20 is $\frac{2}{3}$ of a number, $\frac{1}{2}$ of 20, which is 10, is $\frac{1}{3}$ of that number, and 3 times 10 or 30 must be that number. Since 30 is 6 times $\frac{1}{2}$ of itself, or 5, $\frac{4}{5}$ of 25 is $\frac{2}{3}$ of 6 times 5.

31. $\frac{2}{3}$ of 60 is $\frac{4}{7}$ of 5 times what number?

32. $\frac{3}{5}$ of 90 is $\frac{3}{4}$ of 6 times what number?

33. $\frac{3}{4}$ of 52 is $\frac{3}{8}$ of 4 times what number?

34. $\frac{3}{5}$ of 70 is $\frac{7}{8}$ of 3 times what number?

35. $\frac{5}{6}$ of 72 is $\frac{5}{7}$ of 12 times what number?

36. $\frac{6}{7}$ of 91 is $\frac{3}{4}$ of 8 times what number?

37. $\frac{3}{5}$ of 144 is $\frac{6}{7}$ of 9 times what number?

38. $\frac{4}{5}$ of $\frac{5}{8}$ of 44 is $\frac{5}{9}$ of twice what number?

39. $\frac{2}{3}$ of $\frac{9}{10}$ of 90 is $\frac{9}{10}$ of 12 times what number?

40. $\frac{3}{4}$ of $\frac{7}{12}$ of 96 is $\frac{7}{12}$ of 4 times what number?

41. $\frac{2}{3}$ of John's age is $\frac{5}{6}$ of Mary's age; if John's age is 15 years, how old is Mary?

42. At the battle of Tippecanoe the killed and wounded of Harrison's troops were 180, and $\frac{1}{5}$ of that number is $\frac{10}{10}$ of $\frac{1}{10}$ of the killed and wounded of the Indians. What was the loss of the latter?

43. The towns E, F, and G are in a straight line in the order named. The distance from E to F is 36 miles, and $\frac{3}{4}$ of $\frac{8}{9}$ of this distance is $\frac{1}{4}$ of twice the distance from E to G. How far is it from F to G?

44. Between two towns A and B, the distance is 200 miles, and $\frac{4}{5}$ of this distance is 16 miles less than $\frac{8}{7}$ of twice the distance from C to D. What is the distance from C to D?

45. From a cask of wine were sold 9 gallons more than $\frac{1}{8}$ of the whole, when the remainder was 4 times the quantity sold. How much had the cask at first?

46. A farmer sold 20 acres more than $\frac{1}{3}$ of his farm, and found that the remainder was 20 acres more than $\frac{1}{2}$ of his farm. How large was his farm at first?

47. Mr. Jones is 60 years old, and $\frac{3}{4}$ of his age is $\frac{5}{9}$ of 3 times the age of his son, and $\frac{5}{8}$ of 4 times the age of his daughter. What is the difference between the ages of his son and daughter?

LESSON XXIII.

1. $\frac{2}{3}$ of 18 is $\frac{3}{4}$ of how many fifths of 40?

ANALYSIS.— $\frac{2}{3}$ of 18 is 12: if 12 is $\frac{3}{4}$ of a number, $\frac{1}{3}$ of 12, which is 4, is $\frac{1}{4}$ of that number, and 4 times 4, or 16, must be that number. 16 is as many fifths of 40 as $\frac{1}{5}$ of 40, which is 8, is contained times in 16. Since 8 is contained 2 times in 16. $\frac{2}{3}$ of 18 is $\frac{3}{4}$ of $\frac{2}{5}$ of 40.

2. $\frac{3}{8}$ of 48 is $\frac{2}{3}$ of how many fourths of 36?

3. $\frac{2}{7}$ of 49 is $\frac{7}{12}$ of how many thirds of 36?

4. $\frac{5}{7}$ of 42 is $\frac{3}{8}$ of how many fifths of 100?

5. $\frac{4}{5}$ of 60 is $\frac{3}{4}$ of how many ninths of 72?

6. $\frac{5}{9}$ of 99 is $\frac{1}{2}$ of how many fifths of 75?

7. $\frac{7}{8}$ of 80 is $\frac{1}{5}$ of how many eighths of 120?

8. $\frac{5}{6}$ of 18 is $\frac{3}{10}$ of how many elevenths of 110?

9. $\frac{7}{10}$ of 90 is $\frac{3}{4}$ of how many twelfths of 144?

10. $\frac{8}{11}$ of 132 is $\frac{4}{5}$ of how many tenths of 150?

11. $\frac{5}{12}$ of 108 is $\frac{3}{7}$ of how many ninths of 135?

12. $\frac{3}{4}$ of 32 is $\frac{4}{9}$ of how many sevenths of 126?

13. $\frac{2}{3}$ of $\frac{1}{2}$ of 36 is 3 times what part of 16?

ANALYSIS.— $\frac{1}{2}$ of 36 is 18, and $\frac{2}{3}$ of 18 is 12; if 12 is 3 times a number, $\frac{1}{3}$ of 12, which is 4, is that number. Since 4 is $\frac{1}{4}$ of 16, $\frac{2}{3}$ of $\frac{1}{2}$ of 36 is 3 times $\frac{1}{4}$ of 16.

14. $\frac{2}{5}$ of $\frac{3}{4}$ of 20 is 2 times what part of 27?

15. $\frac{3}{7}$ of $\frac{5}{6}$ of 84 is 3 times what part of 50?

16. $\frac{2}{9}$ of $\frac{3}{4}$ of 144 is 4 times what part of 72?

17. $\frac{5}{8}$ of $\frac{2}{3}$ of 60 is 5 times what part of 30?

18. $\frac{7}{8}$ of $\frac{3}{5}$ of 80 is 6 times what part of 56?

19. $\frac{3}{10}$ of $\frac{5}{12}$ of 168 is 7 times what part of 33?

20. $\frac{8}{11}$ of $\frac{5}{8}$ of 88 is 8 times what part of 45?

21. $\frac{4}{9}$ of $\frac{9}{10}$ of 90 is 9 times what part of 28?

22. $\frac{3}{8}$ of $\frac{5}{11}$ of 176 is 10 times what part of 18?

23. $\frac{1}{2}$ of $\frac{1}{3}$ of 132 is 11 times what part of 10?

24. $\frac{1}{3}$ of $\frac{1}{4}$ of 432 is 12 times what part of 12?

25. $\frac{1}{4}$ of $\frac{1}{5}$ of 110 is 11 times what part of 1?

26. $\frac{1}{5}$ of $\frac{1}{6}$ of 185 is 37 times what part of 1?

27. $\frac{1}{6}$ of $\frac{1}{7}$ of 174 is 29 times what part of 1?

28. $\frac{1}{7}$ of $\frac{1}{8}$ of 175 is 25 times what part of 1?

29. $\frac{3}{10}$ of 100 is $\frac{5}{6}$ of how many times $\frac{4}{5}$ of 15?

ANALYSIS.— $\frac{3}{10}$ of 100 is 30; if 30 is $\frac{5}{6}$ of a number, $\frac{1}{5}$ of 30, which is 6, is $\frac{1}{6}$ of that number, and 6 times 6, which is 36, must be that number. Since $\frac{4}{5}$ of 15 is 12, and 36 is 3 times 12, $\frac{3}{10}$ of 100 is $\frac{5}{6}$ of 3 times $\frac{4}{5}$ of 15.

30. $\frac{2}{3}$ of 72 is $\frac{3}{4}$ of how many times $\frac{2}{3}$ of 24?

31. $\frac{4}{5}$ of 45 is $\frac{9}{10}$ of how many times $\frac{4}{7}$ of 14?

32. $\frac{5}{6}$ of 60 is $\frac{2}{3}$ of how many times $\frac{3}{5}$ of 25?

33. $\frac{3}{7}$ of 56 is $\frac{3}{10}$ of how many times $\frac{2}{7}$ of 35?

34. $\frac{5}{8}$ of 64 is $\frac{4}{9}$ of how many times $\frac{3}{4}$ of 20?

35. $\frac{7}{8}$ of 40 is $\frac{5}{9}$ of how many times $\frac{7}{9}$ of 27?

36. $\frac{4}{9}$ of 63 is $\frac{7}{12}$ of how many times $\frac{2}{5}$ of 30?

37. $\frac{7}{10}$ of 80 is $\frac{1}{2}$ of how many times $\frac{4}{7}$ of 49?

38. $\frac{8}{11}$ of 88 is $\frac{4}{5}$ of how many times $\frac{2}{3}$ of 60?

39. $\frac{7}{12}$ of 144 is $\frac{2}{3}$ of how many times $\frac{7}{10}$ of 20?

40. $\frac{4}{15}$ of 90 is $\frac{3}{7}$ of how many times $\frac{2}{5}$ of 35?

41. A man's barn cost 1000 dollars, and $\frac{4}{5}$ of this sum was $\frac{2}{11}$ of as many times $\frac{1}{8}$ of the cost of his house as there were times 10 acres in his farm. If his house cost 1600 dollars, how many acres of farm had he?

42. Between the towns M and N the distance is 76 miles, and $\frac{3}{4}$ of this distance, increased by 3 miles, is $\frac{5}{6}$ of the distance between N and O diminished by 3 miles. What is the distance from M to O, if the towns are in a straight line in the order M N O?

43. Five-eighths of the number of sheep which A had were 75; but, when he had lost sixteen, and bought $\frac{5}{8}$ as many as he had left, how many more sheep than the original number had he?

44. B bought a suit of clothes, of which the coat cost 24 dollars, and $\frac{2}{3}$ of $\frac{5}{8}$ of this was $\frac{1}{4}$ of the cost of the whole suit. What part of the cost of the coat was the cost of the rest of the suit?

45. C bought a farm for 3600 dollars, and expended for fencing and clearing $\frac{1}{20}$ as much as he gave for the farm. His first crop brought him in $\frac{2}{3}$ of $\frac{6}{20}$ as many dollars as he gave for the farm and improvements. How much did C realize on his first crop?

LESSON XXIV.

REMARK.—A single question can be so changed that every term in it may be made, in its turn, the *required* term. It is a very useful exercise for a pupil to make these changes, giving for every one its proper analysis.

A statement of the question with figures and signs upon the blackboard will be valuable at first, this statement being changed to correspond to the change in the required term. After the pupil has acquired skill in this exercise, he should be able to proceed without the assistance of the blackboard. For example, take the statement 25 is $\frac{5}{8}$ of 4 times 10. This can be written $25 = \frac{5}{8} \times 4 \times 10$. If we make 25 the required number, it may be written $— = \frac{5}{8}$ of 4×10 . This may be read, “What number is $\frac{5}{8}$ of 4 times 10?”

If 5 is the required number, the question may be written $25 = \frac{5}{8} \times 4 \times 10$, and read, “25 is how many eighths of 4 times 10?”

If 8 is the required number, the question may be written $25 = \frac{5}{8} \times 4 \times 10$, and read, “25 is 5 times what part of 4 times 10?”

If 4 is the required number, the question may be written $25 = \frac{5}{8} \times — \times 10$, and read, “25 is $\frac{5}{8}$ of how many times 10?”

If 10 is the required number, the question may be written $25 = \frac{5}{8} \times 4 \times —$, and read, “25 is $\frac{5}{8}$ of 4 times what number?”

These changes can, each in turn, be made by erasing a single term of the expression, and, after the analysis has been made, the answer can be put into the blank to complete the expression.

$$1. — = \frac{5}{12} \times 16 \times 3?$$

$$2. 20 = \frac{5}{12} \times 16 \times 3?$$

$$3. 20 = \frac{5}{12} \times 16 \times 3?$$

$$4. 20 = \frac{5}{12} \times — \times 3?$$

$$5. 20 = \frac{5}{12} \times 16 \times —?$$

$$6. \frac{5}{8} \times 72 = \frac{3}{4} \times 60?$$

$$7. \frac{5}{8} \times 72 = \frac{3}{4} \times 60?$$

$$8. \frac{5}{8} \times — = \frac{3}{4} \times 60?$$

$$9. \frac{5}{8} \times 72 = \frac{3}{4} \times 60?$$

10. $\frac{5}{8} \times 72 = \frac{3}{4} \times 60?$
11. $\frac{5}{8} \times 72 = \frac{3}{4} \times -?$
12. $\frac{5}{9} \times 99 = \frac{1\frac{1}{2}}{1} \times \frac{4}{5} \times 75?$
13. $\frac{5}{9} \times 99 = \frac{1\frac{1}{2}}{1} \times \frac{4}{5} \times 75?$
14. $\frac{5}{9} \times - = \frac{1\frac{1}{2}}{1} \times \frac{4}{5} \times 75?$
15. $\frac{5}{9} \times 99 = \frac{1\frac{1}{2}}{1} \times \frac{4}{5} \times 75?$
16. $\frac{5}{9} \times 99 = \frac{1\frac{1}{2}}{1} \times \frac{4}{5} \times 75?$
17. $\frac{5}{9} \times 99 = \frac{1\frac{1}{2}}{1} \times \frac{4}{5} \times 75?$
18. $\frac{5}{9} \times 99 = \frac{1\frac{1}{2}}{1} \times \frac{4}{5} \times -?$
19. $\frac{5}{9} \times 99 = \frac{1\frac{1}{2}}{1} \times \frac{4}{5} \times -?$
20. $\frac{2}{3} \times \frac{9}{10} \times 90 = \frac{3}{4} \times 12 \times 6?$
21. $\frac{2}{3} \times \frac{9}{10} \times 90 = \frac{3}{4} \times 12 \times 6?$
22. $\frac{2}{3} \times \frac{9}{10} \times 90 = \frac{3}{4} \times 12 \times 6?$
23. $\frac{2}{3} \times \frac{9}{10} \times 90 = \frac{3}{4} \times 12 \times 6?$
24. $\frac{2}{3} \times \frac{9}{10} \times - = \frac{3}{4} \times 12 \times 6?$
25. $\frac{2}{3} \times \frac{9}{10} \times 90 = \frac{3}{4} \times 12 \times 6?$
26. $\frac{2}{3} \times \frac{9}{10} \times 90 = \frac{3}{4} \times 12 \times 6?$
27. $\frac{2}{3} \times \frac{9}{10} \times 90 = \frac{3}{4} \times - \times 6?$
28. $\frac{2}{3} \times \frac{9}{10} \times 90 = \frac{3}{4} \times 12 \times -?$
29. $\frac{2}{3} \times 72 = \frac{3}{4} \times 4 \times \frac{2}{3} \times 24?$
30. $\frac{2}{3} \times 72 = \frac{3}{4} \times 4 \times \frac{2}{3} \times 24?$
31. $\frac{2}{3} \times - = \frac{3}{4} \times 4 \times \frac{2}{3} \times 24?$
32. $\frac{2}{3} \times 72 = \frac{3}{4} \times 4 \times \frac{2}{3} \times 24?$
33. $\frac{2}{3} \times 72 = \frac{3}{4} \times 4 \times \frac{2}{3} \times 24?$
34. $\frac{2}{3} \times 72 = \frac{3}{4} \times - \times \frac{2}{3} \times 24?$
35. $\frac{2}{3} \times 72 = \frac{3}{4} \times 4 \times \frac{2}{3} \times 24?$
36. $\frac{2}{3} \times 72 = \frac{3}{4} \times 4 \times \frac{2}{3} \times 24?$
37. $\frac{2}{3} \times 72 = \frac{3}{4} \times 4 \times \frac{2}{3} \times -?$
38. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times 3 \times \frac{7}{9} \times 27?$
39. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times 3 \times \frac{7}{9} \times 27?$
40. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times 3 \times \frac{7}{9} \times 27?$
41. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times 3 \times \frac{7}{9} \times 27?$
42. $\frac{4}{5} \times \frac{7}{8} \times - = \frac{4}{9} \times 3 \times \frac{7}{9} \times 27?$
43. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times 3 \times \frac{7}{9} \times 27?$
44. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times 3 \times \frac{7}{9} \times 27?$
45. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times - \times \frac{7}{9} \times 27?$
46. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times 3 \times \frac{7}{9} \times 27?$
47. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times 3 \times \frac{7}{9} \times 27?$
48. $\frac{4}{5} \times \frac{7}{8} \times 40 = \frac{4}{9} \times 3 \times \frac{7}{9} \times -?$

REMARK.—The teacher can, from time to time, make examples for class exercise like the above, to give variety and interest to the labors of the classes in the following parts of the book.

LESSON XXV.

1. In the number 5555, what part of the 5 *thousands* are the 5 *hundreds*?

ANS. 5 *hundreds* are $\frac{1}{10}$ of 5 *thousands*, because a hundred is $\frac{1}{10}$ of a thousand.

2. What part of the 5 *hundreds* are the 5 *tens*?

ANS. 5 *tens* are $\frac{1}{10}$ of 5 *hundreds*, because ten is $\frac{1}{10}$ of one hundred.

3. What part of the 5 *tens* are the 5 *units*?

ANS. 5 *units* are $\frac{1}{10}$ of 5 *tens*, because 1 is $\frac{1}{10}$ of 10.

4. What part of 5 *units* are 5-tenths?

ANS. 5-tenths are $\frac{1}{10}$ of 5 *units*, because 1-tenth is $\frac{1}{10}$ of 1, or $\frac{1}{10}$ of a unit.

5. In writing $55\frac{5}{10}$, how else can the fractional part of the number be written?

ANS. The fraction could be written without its denominator 10, because the place of its numerator 5, at the right of units, would make it mean 5-tenths. To distinguish the fraction from the whole number, a point, called a *separatrix*, is put between them. Thus $55.5 = 55\frac{5}{10}$.

6. What does the next right-hand place mean?

ANS. *Tenths* of *tenths*, or *hundredths*, because $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$. Therefore 55.55 means $55\frac{5}{10} + \frac{5}{100}$.

7. How much are $\frac{5}{10} + \frac{5}{100}$?

ANS. $\frac{5}{10}$ are $\frac{50}{100}$, and $\frac{50}{100} + \frac{5}{100} = \frac{55}{100}$. Therefore 55.55 means $55\frac{55}{100}$.

8. What does the next right-hand place mean?

ANS. *Tenths* of *hundredths*, or *thousandths*, because $\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000}$. Therefore 55.555 means $55\frac{55}{100} + \frac{5}{1000}$.

9. How much are $\frac{55}{100} + \frac{5}{1000}$?

ANS. $\frac{55}{100} = \frac{550}{1000}$, and $\frac{550}{1000} + \frac{5}{1000} = \frac{555}{1000}$. Therefore 55.555 means $55\frac{555}{1000}$.

10. What does the next right-hand place mean?

ANS. *Tenths* of *thousandths*, or *ten-thousandths*.

11. What are such fractions called?

ANS. *Decimal* fractions, because they proceed by *tenths*, like the figures in whole numbers.

12. How can $\frac{5}{100}$ be decimally written?

ANS. 5 must be in the *hundredths* place, thus, .05.

13. How can $\frac{9}{1000}$ be decimally written?

ANS. 9 must be in the *thousandths* place, thus, .009.

14. How can $\frac{23}{100}$ be decimally written?

ANS. 23 must be so written that the 3 shall be in the *hundredths* place, thus, .23.

15. How can $\frac{34}{1000}$ be decimally written?

ANS. 34 must be so written that the 4 shall be in the *thousandths* place, thus, .034.

16. How can $\frac{859}{100}$ be written as a mixed decimal?

ANS. The last figure, 9, must be written in the *hundredths* place, thus, 8.59.

17. Reduce $\frac{1}{2}$ to a decimal.

ANS. Multiplying both terms by 5 to convert the denominator to 10, $\frac{1}{2} = \frac{5}{10}$, or, decimally, .5.

18. Reduce to a decimal $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

19. How can this result be easily produced?

ANS. By annexing 0's to the numerator, and dividing by the denominator.

20. What is $\frac{3}{10}$ of 15?

ANS. $\frac{3}{10}$ of 15 is $\frac{45}{10}$, equal to $4\frac{5}{10}$, or 4.5.

21. How can this be done decimally?

ANS. $15 \times .3 = 4.5$.

22. What, then, is the rule for multiplying by decimals?

ANS. *Multiply as with whole numbers, and point off as many figures from the right of the product, for decimals, as there are decimal figures in both factors.*

23. Find $\frac{7}{10}$ of 25 by analysis, and by decimals.

24. Find $\frac{9}{10}$ of 21 by analysis, and by decimals.

25. Find $\frac{5}{10}$ of $4\frac{3}{10}$ by analysis, and by decimals.

26. Find $\frac{12}{100}$ of $5\frac{5}{10}$ by analysis, and by decimals.

27. Find $\frac{6}{100}$ of $9\frac{75}{100}$ by analysis, and by decimals.

28. Find $\frac{3}{1000}$ of 100 by analysis, and by decimals.

29. Find $\frac{4}{5}$ of $\frac{10}{20}$ by analysis, and by decimals.

CHAPTER VIII.

EXERCISES IN REDUCTION.

LESSON I.

UNITED STATES MONEY.

10 mills (marked m.)	make	1 cent,	marked	ct.
10 cents	"	1 dime,	"	d.
10 dimes	"	1 dollar,	"	\$.
10 dollars	"	1 eagle,	"	E.

1. How many mills are there in 2 cts.? 3 cts.? $\frac{1}{2}$ ct.? 5 cts.? 6 cts.? 7 cts.? 8 cts.? 9 cts.?

2. How many mills are there in $\frac{1}{2}$ ct.? $\frac{1}{3}$ ct.? $\frac{1}{4}$ ct.? $\frac{1}{5}$ ct.? $\frac{2}{3}$ ct.? $\frac{3}{4}$ ct.? $\frac{2}{5}$ ct.? $\frac{3}{5}$ ct.? $\frac{4}{5}$ ct.? $\frac{7}{10}$ ct.?

3. How many mills are there in $1\frac{1}{2}$ cts.? $2\frac{1}{4}$ cts.? $3\frac{9}{10}$ cts.? $4\frac{3}{5}$ cts.? $5\frac{2}{3}$ cts.? $6\frac{7}{10}$ cts.? 10 cts.?

4. How many cents are there in 1 dime? $1\frac{1}{2}$ d.? $2\frac{1}{2}$ d.? $3\frac{3}{4}$ d.? $6\frac{1}{4}$ d.? $7\frac{1}{2}$ d.? $8\frac{1}{5}$ d.? $9\frac{4}{5}$ d.? 10 d.?

5. How many mills are there in 1 dime? $1\frac{1}{2}$ d.? $2\frac{1}{2}$ d.? $6\frac{1}{4}$ d.? $3\frac{3}{4}$ d.? $7\frac{1}{2}$ d.? $8\frac{1}{5}$ d.? $9\frac{4}{5}$ d.? 10 d.?

6. How many dimes are there in \$1? $\$2\frac{1}{2}$? $\$3\frac{1}{4}$? $\$4\frac{3}{4}$? $\$5\frac{1}{5}$? $\$6\frac{2}{5}$? $\$7\frac{3}{5}$? $\$9\frac{1}{8}$? $\$10\frac{5}{8}$?

7. How many cents are there in \$1? $\$2\frac{1}{2}$? $\$3\frac{1}{4}$? $\$4\frac{3}{4}$? $\$5\frac{1}{5}$? $\$6\frac{2}{5}$? $\$7\frac{3}{5}$? $\$9\frac{1}{8}$? $\$10\frac{5}{8}$?

8. How many mills are there in \$1? $\$2\frac{1}{2}$? $\$3\frac{1}{4}$? $\$4\frac{3}{4}$? $\$5\frac{1}{5}$? $\$6\frac{2}{5}$? $\$7\frac{3}{5}$? $\$9\frac{1}{8}$? $\$10\frac{5}{8}$?

9. How many dollars are there in 1 eagle? 2 E.? $2\frac{1}{2}$ E.? 5 E.? $7\frac{1}{2}$ E.? $6\frac{1}{8}$ E.? 10 E.? $3\frac{3}{4}$ E.?

NOTE.—Because cents are *hundredths* of a dollar, they are usually written decimally as *hundredths* at the right of the number of dollars; and because mills are *tenths* of a cent, they are usually written decimally at the right of cents, or in the third place from dollars, because they are thousandths of a dollar. Thus 2 dollars, 34 cents, 5 mills are written \$2.34, 5, or \$2.345.

10. How many cents in \$2.34? In \$5.48?
11. How many cents in \$14.50? In \$18.75?
12. How many mills in \$1.45, 6? In \$7.68, 2?
13. How many mills in \$.57, 9? In \$.75, 3?
14. Express both as a common fraction and as a decimal, what part of a dollar 5 cts. are; 6 cts.; 7 cts.; 8 cts.; 4 cts.; 25 cts.; 50 cts.; 75 cts.; 20 cts.; 30 cts.; 80 cts.; 95 cts.
15. Express as a common fraction, and as a decimal, what part of a cent 5 m. are; 2 m.; 3 m.; 4 m.; 6 m.; 7 m.; 8 m.; 9 m.; $2\frac{1}{2}$ m.; $7\frac{1}{2}$ m.; $1\frac{1}{4}$ m.
16. How many cents are there in 75 m.? $87\frac{1}{2}$ m.? $62\frac{1}{2}$ m.? 140 m.? 250 m.? 625 m.? 875 m.? 95 m.? 375 m.?
17. How many dollars are there in 750 cts.? 125 cts.? 375 cts.? $487\frac{1}{2}$ cts.? $237\frac{1}{2}$ cts.? 520 cts.? 606 cts.?
18. How many dollars are there in 2000 m.? 3000 m.? 1500 m.? 2500 m.? 4560 m.? 6785 m.?

ENGLISH MONEY.

4 farthings (qr. or f.)	make 1 penny,	(d.)
12 pence	“ 1 shilling,	(s.)
20 shillings	“ 1 pound or sovereign,	(£)
21 shillings	“ 1 guinea,	(guin.)
5 shillings	“ 1 crown,	

1. How many qr. in $2\frac{1}{2}$ d.? $3\frac{1}{4}$ d.? $5\frac{3}{4}$ d.? $12\frac{1}{2}$ d.?
2. How many d. in $5\frac{1}{2}$ s.? $6\frac{1}{4}$ s.? $4\frac{3}{4}$ s.? $8\frac{5}{8}$ s.?
3. How many s. in $2\frac{1}{2}$ £? $5\frac{1}{4}$ £? $3\frac{3}{4}$ £? $6\frac{1}{5}$ £?
4. How many f. in £1 1 s. 1 d. 1 f.?
5. What part of £2 is 8 s.? Of 5 s. is 4 d.?
6. How many d. in 18 f.? How many s. in 168 f.?
7. How many £ in 85 s.? 360 d.? 1200 f.?

LESSON II.

LONG OR LINEAR MEASURE.

12 inches (in.)	make 1 foot,	(ft.)
3 feet	“ 1 yard,	(yd.)
$5\frac{1}{2}$ yds., or $16\frac{1}{2}$ ft.	“ 1 rod, pole, or perch,	(rd.)
40 rods	“ 1 furlong,	(fur.)
8 fur., or 320 rods	“ 1 mile,	(m.)
3 miles	“ 1 league,	(lea.)

1. How many inches in $2\frac{1}{2}$ ft.? $1\frac{1}{4}$ yd.? 2 rd.?
2. How many ft. in $3\frac{2}{3}$ yd.? 4 rd.? 1 fur.?
3. How many yd. in 6 rd.? $\frac{1}{4}$ fur.? $2\frac{1}{2}$ rd.?
4. How many inches in 1 yd. 2 ft. 6 in.?
5. How many rd. in $1\frac{1}{2}$ fur.? $3\frac{1}{4}$ fur.? $4\frac{5}{8}$ fur.? $2\frac{7}{10}$ fur.?
6. How many rd. in $\frac{1}{8}$ m.? $\frac{3}{10}$ m.? 1 lea.?
7. How many fur. in 1 lea.? $2\frac{1}{2}$ m.? $5\frac{3}{4}$ m.?
8. How many ft. in 48 in.? 72 in.? 108 in.? 138 in.?
9. How many yd. in $31\frac{1}{2}$ ft.? 35 ft.? 48 ft.? 72 ft.?
10. How many m. in 480 rd.?
11. What part of a mile are 40 rd.? 80 rd.? 200 rd.?
12. What part of a yd. are 6 in.? 9 in.? 12 in.? 15 in.? 16 in.? 18 in.? 20 in.? 21 in.? 24 in.? 27 in.?
13. How many ft. in 1 rd. 2 yd. $1\frac{1}{2}$ ft.?

ENGINEERS' AND SURVEYORS' MEASURE.

$7\frac{92}{100}$ inches (in.)	make 1 link,	(l.)
25 links	“ 1 pole or rod,	(p.)
4 poles, or 66 ft.	“ 1 chain,	(ch.)
10 chains	“ 1 furlong,	(fur.)
8 furlongs, or 80 chains	“ 1 mile,	(m.)

1. How many p. in $62\frac{1}{2}$ l.? 75 l.? $81\frac{1}{4}$ l.? 100 l.?
2. How many l. in $5\frac{1}{2}$ p.? 6 p.? $3\frac{1}{4}$ p.? $1\frac{3}{4}$ p.?

3. How many ft. in 2 ch. ? $3\frac{1}{2}$ ch. ? $2\frac{2}{3}$ ch. ?
4. How many p. in $4\frac{1}{4}$ ch. ? $5\frac{1}{2}$ ch. ? $6\frac{3}{4}$ ch. ?
5. How many ch. in 10 p. ? 15 p. ? 29 p. ? 80 p. ?
6. How many ch. in $1\frac{1}{4}$ m. ? $2\frac{1}{2}$ m. ? $3\frac{3}{4}$ m. ?
7. How many miles in 120 ch. ? 360 ch. ? 40 ch. ?
8. What part of a mile are 10 ch. ? 16 ch. ? 20 ch. ?
25 ch. ? 30 ch. ? 60 ch. ? 64 ch. ?
9. What part of a chain are 30 l. ? 45 l. ? 50 l. ? $62\frac{1}{2}$ l. ? 75 l. ? 80 l. ? 96 l. ?
10. How many poles in 1 m. 5 ch. 50 l. ?

CLOTH MEASURE.

$2\frac{1}{4}$ inches (in.)	make	1 nail,	(na.)
4 nails, or 9 inches	“	1 quarter of a yard,	(qr.)
4 quarters	“	1 yard,	(yd.)
3 qr.	“	1 Flemish ell,	(Fl. E.)
4 qr. $1\frac{1}{5}$ in.	“	1 Scotch ell,	(S. E.)
5 qr.	“	1 English ell,	(E. E.)
6 qr.	“	1 French ell,	(Fr. E.)

1. How many inches in 1 yd. ? $1\frac{1}{4}$ yd. ? $2\frac{1}{2}$ yd. ? $3\frac{3}{4}$ yd. ?
2. How many inches in 1 Fl. E. ? 1 S. E. ? 1 E. E. ? 1 Fr. E. ?
3. How many qr. in $1\frac{1}{4}$ yd. ? $2\frac{1}{2}$ yd. ? $5\frac{1}{4}$ yd. ?
4. How many qr. in 2 Fl. E. ? $3\frac{2}{3}$ E. E. ? $4\frac{1}{2}$ Fr. E. ?
5. How many yards in 48 in. ? 90 in. ? 108 in. ? 120 in. ?
6. How many yards in 7 qr. ? 9 qr. ? 26 qr. ? 35 qr. ?
7. How many yards in 4 Fl. E. ? 4 E. E. ? 4 Fr. E. ?
8. What part of a yard is a Flemish ell ?
9. What part of an English ell is a yard ?
10. What part of a French ell is a yard ?
11. What cost $2\frac{3}{4}$ yd. of cloth at $\$2\frac{3}{4}$ a yd. ?
12. What cost $1\frac{1}{2}$ quarters of silk at \$1 a yard ?
13. Bought 40 E. E. of broadcloth at \$5 per ell, and sold it at \$5 per yard ; what was the gain ?
14. Bought 60 Fr. E. of cassimere at \$1.50 per ell, and sold it at the rate of \$1 per Fl. E. ; what did I receive for the cloth, and how much did I gain ?

LESSON III.

SURFACE OR SQUARE MEASURE.

A *square inch* is a square surface 1 inch long and 1 inch broad.

A *square foot* is a square surface 1 foot long and 1 foot broad.

A *square yard* is a square surface 1 yard long and 1 yard broad.

A *square rod* is a square surface 1 rod long and 1 rod broad.

A *square mile* is a square surface 1 mile long and 1 mile broad.

144 square inches (sq. in.)	make 1 square foot, (sq. ft.)
9 square feet	" 1 square yard, (sq. yd.)
30 $\frac{1}{4}$ square yards, or }	" 1 square rod, }
272 $\frac{1}{4}$ square feet }	or pole, or perch, } (P.)
40 square rods	" 1 rood, (R.)
4 roods, or 10 sq. ch.	" 1 acre, (A.)
640 acres	" 1 square mile. (sq. m.)

1. If you divide a square foot into square inches, how many rows would there be, and how many square inches in each row? How many in all?

2. Show, in the same manner, that 9 sq. ft. make 1 sq. yd.

3. A square rod is 5 $\frac{1}{2}$ yds. long and 5 $\frac{1}{2}$ yds. broad; from this prove that 30 $\frac{1}{4}$ sq. yds. make 1 P.

4. A square rod is 16 $\frac{1}{2}$ ft. long and 16 $\frac{1}{2}$ ft. broad; from this prove that 272 $\frac{1}{4}$ sq. ft. make 1 P.

5. How many sq. in. in 3 sq. ft.? In a surface 3 feet square?

6. How many P. in 1 A.? 2 $\frac{1}{2}$ A.? 3 $\frac{1}{4}$ A.?

7. How many A. in 480 P.? 400 P.? 1000 P.?

8. How many sq. yds. in 67 $\frac{1}{2}$ sq. ft.? 42 $\frac{3}{4}$ sq. ft.?

9. How many acres in 2 sq. m.? 2 miles square?

NOTE.—The quantity of any square surface is found by multiplying its length by its breadth, expressed in like units.

10. How many acres in a farm, 320 rods long and 80 rods broad, having straight sides and square corners?

11. How many sq. ft. in a carpet 15 ft. long and 12 ft. broad? How many square yards?

12. How many square miles in a township $5\frac{1}{2}$ miles long and $7\frac{1}{2}$ miles broad?

SOLID OR CUBIC MEASURE.

A *cube* is a body having six square surfaces.

A *cubic inch* is a cube, whose edges are an inch long.

A *cubic foot* is a cube, whose edges are a foot long.

A *cubic yard* is a cube, whose edges are a yard long.

1728 cubic inches (cu. in.)	make 1 cubic foot, (cu. ft.)
27 cubic feet	“ 1 cubic yard, (cu. yd.)
128 cubic feet	“ 1 cord of wood, (C.)
42 cubic feet	“ 1 ton of shipping.

NOTE.—To find the cubical quantity of any cubic thing, multiply its length by its breadth, and that product by its thickness.

1. Prove that 1728 cu. in. make a cu. ft.

2. Prove that 27 cu. ft. make a cu. yd.

3. How many cubic inches in 3 cu. ft.? 9 cu. ft.?

4. How many cubic feet in 2 cu. yds.? 10 cu. yds.?

5. How many cubic inches in a board 12 inches long, 7 inches broad, and $1\frac{1}{4}$ inches thick?

6. How many cubic feet of earth must be dug to make a cellar 50 ft. long, 20 ft. broad, and 6 ft. deep?

7. How much wood is in a pile 8 feet long, 4 feet wide, and 4 feet high?

8. How many cords of wood can be piled in a shed 16 feet long, 12 feet wide, and 8 feet high?

9. How many cubic yards of air in a room 15 feet long, 12 feet wide, and 9 feet high?

10. How many cu. yds. in 36 cu. ft.?

LESSON IV.

AVOIRDUPOIS WEIGHT.

Avoirdupois Weight is used in weighing almost all articles taken in large quantity, such as groceries, &c.

16 drams (dr.)	make 1 ounce,	(oz.)
16 ounces	“ 1 pound,	(lb.)
25 pounds	“ 1 quarter,	(qr.)
4 quarters, or 100 lbs.,	“ 1 hundredweight,	(cwt.)
20 cwt., or 2000 lbs.,	“ 1 ton.	(T.)

1. How many dr. in 5 oz.? 7 oz.? $8\frac{1}{2}$ oz.? $10\frac{1}{4}$ oz.?
2. How many oz. in $2\frac{1}{2}$ lb.? $3\frac{1}{4}$ lb.? 5 lb. 2 oz.?
3. How many lb. in 2 cwt. 3 qr. 20 lb.?
4. How many lb. in 2 T. 4 cwt. 1 qr. 12 lb.?
5. How many lb. in 72 oz.? 84 oz.? 98 oz.? 124 oz.?
6. How many cwt. in 250 lb.? 325 lb.? 575 lb.?
7. How many T. in 45 cwt.? 70 cwt.? 115 cwt.?

TROY WEIGHT.

Troy Weight is used in weighing coins, precious metals, jewels, and liquors.

24 grains (gr.)	make 1 pennyweight, (pwt.)
20 pennyweights	“ 1 ounce, (oz.)
12 ounces	“ 1 pound, (lb.)

The ounce and pound of Troy Weight are not the same weights as the ounce and pound Avoirdupois.

1. How many gr. in $3\frac{1}{2}$ pwt.? $4\frac{1}{2}$ pwt.? $5\frac{1}{4}$ pwt.?
2. How many gr. in 1 oz.? 2 oz.? 3 oz.? 4 oz.?
3. How many pwt. in 1 lb.? 84 gr.? $2\frac{1}{2}$ lb.? 100 gr.?
4. How many oz. in 50 pwt.? 125 pwt.? 275 pwt.?
5. How many lb. in 30 oz.? 40 oz.? 81 oz.? 50 oz.?

APOTHECARIES' WEIGHT.

Apothecaries' Weight is used in medical prescriptions.

20 grains (gr.)	make 1 scruple,	(\mathfrak{D})
3 scruples	“ 1 dram,	(\mathfrak{z})
8 drams	“ 1 ounce,	(\mathfrak{z})
12 ounces	“ 1 pound,	(\mathfrak{lb})

In this weight the grain, ounce, and pound are the same as in Troy Weight.

1. How many gr. in $2\frac{1}{2} \mathfrak{D}$? $3\frac{3}{4} \mathfrak{D}$? $4\frac{4}{5} \mathfrak{D}$? $5 \mathfrak{D}$? $10 \mathfrak{D}$?
2. How many \mathfrak{D} in $2\frac{1}{3} \mathfrak{z}$? $4\frac{2}{3} \mathfrak{z}$? $8 \mathfrak{z}$? $10 \mathfrak{z}$? $20 \mathfrak{z}$?
3. How many \mathfrak{z} in $2\frac{3}{4} \mathfrak{lb}$? $4\frac{1}{2} \mathfrak{lb}$? $5\frac{5}{8} \mathfrak{lb}$? $10 \mathfrak{lb}$? $12 \mathfrak{lb}$?
4. How many \mathfrak{z} in $2\frac{3}{4} \mathfrak{lb}$? $5\frac{1}{2} \mathfrak{lb}$? $3\frac{5}{8} \mathfrak{lb}$? $10 \mathfrak{lb}$?
5. How many \mathfrak{lb} in $42 \mathfrak{z}$? $51 \mathfrak{z}$? $64 \mathfrak{z}$? $93 \mathfrak{z}$?
6. How many \mathfrak{z} in $50 \mathfrak{z}$? $60 \mathfrak{z}$? $78 \mathfrak{z}$? $85 \mathfrak{z}$?
7. How many \mathfrak{z} in $19 \mathfrak{D}$? $35 \mathfrak{D}$? $48 \mathfrak{D}$? $60 \mathfrak{D}$?
8. How many \mathfrak{D} in 44 gr. ? 65 gr. ? 70 gr. ? 95 gr. ?
9. How many gr. in $1 \mathfrak{z}$ $1 \mathfrak{D}$ $1 \mathfrak{z}$ $1 \mathfrak{D}$ 1 gr. ?

APOTHECARIES' FLUID MEASURE.

Apothecaries' Fluid Measure is used in measuring the fluid portions of medical prescriptions.

60 minims (\mathfrak{M})	make 1 fluidrachm,	(\mathfrak{fz})
8 fluidrachms	“ 1 fluidounce,	(\mathfrak{fz})
16 fluidounces	“ 1 pint,	(\mathfrak{O} .)
8 pints	“ 1 gallon.	(Cong.)

1. How many minims in $2\frac{1}{2} \mathfrak{fz}$? $3\frac{1}{4} \mathfrak{fz}$? $1\frac{3}{4} \mathfrak{fz}$?
2. How many \mathfrak{fz} in $1\frac{1}{2} \mathfrak{fz}$? $3\frac{1}{2} \mathfrak{fz}$? $5\frac{1}{4} \mathfrak{fz}$? $6\frac{3}{4} \mathfrak{fz}$?
3. How many \mathfrak{fz} in $1\frac{1}{2}$ pints? 2 pints? 3 pints?
4. How many pints in 4 gallons? $10\frac{1}{2}$ gallons?
5. How many pints in $56 \mathfrak{fz}$? $68 \mathfrak{fz}$? $92 \mathfrak{fz}$?
6. How many \mathfrak{fz} in $36 \mathfrak{fz}$? $50 \mathfrak{fz}$? $78 \mathfrak{fz}$?

LESSON V.

LIQUID OR WINE MEASURE.

4 gills (gi.)	make	1 pint,	(pt.)
2 pints	"	1 quart,	(qt.)
4 quarts	"	1 gallon,	(gal.)

NOTE.—The gallon of Wine Measure has a capacity of 231 cubic inches; that of Beer Measure 282 cubic inches. Beer Measure is sometimes used for the measurement of beer, ale, and milk. All other liquids are measured by Wine Measure. The English gallon is the same for all liquids, having a capacity of $277\frac{274}{1000}$ cubic inches.

1. How many gills in $1\frac{1}{2}$ pints? $5\frac{1}{4}$ pts.? $6\frac{3}{4}$ pts.?
2. How many gills in $1\frac{1}{2}$ qts.? $3\frac{1}{2}$ qts.? 1 gallon?
3. How many pints in 5 qts.? 6 qts.? 5 gallons?
4. How many quarts in $1\frac{1}{2}$ gallons? $6\frac{3}{4}$ gallons?
5. How many pints in a barrel holding 40 gallons? In a barrel holding $31\frac{1}{2}$ gallons? 36 gallons?
6. How many gallons of wine in 3 pipes, each containing 126 gallons?
7. How many quarts of oil in a hogshead containing 63 gallons? In a hogshead containing 85 gallons?
8. How many qts. in 25 pts.? How many gallons?

DRY MEASURE.

2 pints (pt.)	make	1 quart,	(qt.)
8 quarts	"	1 peck,	(pk.)
4 pecks	"	1 bushel,	(bu.)

NOTE.—The bushel in the U.S. has a capacity of $2150\frac{42}{100}$ cu. in. That of Great Britain has a capacity of $2218\frac{122}{1000}$ cu. in.

1. How many pints in 5 qts. 1 pt.? In 1 peck?
2. How many qts. in $2\frac{1}{2}$ pk.? In 1 bushel?
3. How many pecks in $3\frac{1}{4}$ bu.? $5\frac{1}{2}$ bu.? $8\frac{3}{4}$ bu.?
4. How many pecks in 18 qts.? 33 qts.? 22 qts.?

LESSON VI.

ANGULAR OR CIRCULAR MEASURE.

Portions of the circumference of a circle are expressed as parts of the whole circumference, without reference to the absolute length of those portions.

60 seconds (")	make	1 minute,	(')
60 minutes	"	1 degree,	(°)
360 degrees	"	1 circumference,	(C.)

1. How many " in $1' 30''$? In $2' 45''$?
2. How many " in $1^\circ 1' 50''$? In $10'$?
3. How many ' in 3° ? $4\frac{1}{2}^\circ$? $5\frac{3}{4}^\circ$? $2\frac{3}{5}^\circ$? $1\frac{2}{3}^\circ$?
4. How many ' in $1^\circ 45'$? In $3^\circ 30'$?
5. How many ° in $200'$? $300'$? $500'$?
6. How many ° in $\frac{1}{4}$ C.? $\frac{1}{3}$ C.? $\frac{4}{5}$ C.? $\frac{5}{6}$ C.? $\frac{3}{10}$ C.?
7. What part of a C. are 20° ? 30° ? 45° ? 90° ? 180° ?

MEASURE OF TIME.

60 seconds (sec.)	make	1 minute,	(min.)
60 minutes	"	1 hour,	(hr.)
24 hours	"	1 day,	(da.)
7 days	"	1 week,	(wk.)
12 calendar months	"	1 year,	(yr.)
365 days (or 52 wk. 1 day)	"	1 common year,	(c. yr.)
366 days (or 52 wk. 2 days)	"	1 leap year,	(l. yr.)
365 da. 5 hr. 48 min. $49\frac{7}{10}$ sec.	"	1 solar year.	
100 years	"	1 century.	

The Earth turns from west to east one complete revolution in 24 hours, thus making one day.

The earth moves around the Sun one complete circuit in 365 da. 5 hr. 48 min. $49\frac{7}{10}$ sec., thus making a year.

Calendar months are those months which are called by distinct names in a calendar. They are twelve, and their

names are January, February, March, April, May, June, July, August, September, October, November, December.

The calendar months have days as follows :—

January,	or	1st month,	has 31 days.
February,	or	2d month,	has 28 days; in leap-year 29.
March,	or	3d month,	has 31 days.
April,	or	4th month,	has 30 days.
May,	or	5th month,	has 31 days.
June,	or	6th month,	has 30 days.
July,	or	7th month,	has 31 days.
August,	or	8th month,	has 31 days.
September,	or	9th month,	has 30 days.
October,	or	10th month,	has 31 days.
November,	or	11th month,	has 30 days.
December,	or	12th month,	has 31 days.

Commit to memory the following lines :—

Thirty days have September,
 April, June, and November,
 And all the rest have thirty-one,
 Save February, which alone
 Has twenty-eight; and we assign
 To this, in leap-year, twenty-nine.

Or,

The fourth, eleventh, ninth, and sixth,
 Have thirty days to each affixed;
 And all the rest have thirty-one,
 Except the second month alone,
 To which we twenty-eight assign,
 Till leap-year gives it twenty-nine.

1. How many sec. in $2\frac{1}{2}$ min. ? $3\frac{3}{4}$ min. ? 5 min. ?
2. How many min. in 10 hr. ? $3\frac{1}{2}$ hr. ? $4\frac{3}{4}$ hr. ? 24 hr. ?
3. How many hr. in $1\frac{1}{4}$ da. ? $2\frac{1}{2}$ da. ? $3\frac{2}{3}$ da. ? $4\frac{5}{6}$ da. ?
4. How many days in 3 wk. ? $5\frac{2}{7}$ wk. ? 10 wk. ? 12. wk. ?
5. How many weeks in the first three months of the c. yr. ? The 2d three ? The 3d three ? The 4th three ?
6. How many hours in the month of June ?
7. How many minutes in 1 wk. 1 da. 8 hr. ?

8. How many days from Jan. 5 to Mar. 17 inclusive, in leap-year? From Apr. 3d to July 4th inclusive?

9. How many days from Aug. 11th to Nov. 29th inclusive?

10. How many weeks from Dec. 7 to Feb. 21st inclusive?

11. If Friday is Feb. 1, what day is Mch. 11 in l. yr.?

12. If Jan. 1 is Monday, what day is Aug. 1 in l. yr.?

13. If June 6 is Tuesday, what day is Nov. 11?

14. If Thurs. is Dec. 1, what is the last day of Dec.?

MISCELLANEOUS TABLE.

12 things	make	1 dozen.
12 dozen, or 144 things,	"	1 gross.
12 gross, or 1728 things,	"	1 great gross.
20 things	"	1 score.
24 sheets of paper	"	1 quire.
20 quires, or 480 sheets,	"	1 ream.
2 reams	"	1 bundle.
5 bundles, or 10 reams,	"	1 bale.
100 pounds of fish	"	1 quintal.
196 pounds of flour	"	1 barrel.

1. What cost 1 gross of buttons at $\$ \frac{3}{8}$ a dozen?

2. What cost 2 gross of pens at 4 cents a dozen?

3. How many score years has a person lived, who is 60 years old? 70 years old? 80 years old?

4. How many score make a hundred?

5. How many sheets of paper in a half-quire? $2\frac{1}{2}$ quires? $\frac{1}{4}$ ream? $\frac{1}{2}$ ream? $\frac{3}{4}$ ream?

6. Bought a ream of paper for \$2; what was the cost per quire?

7. Bought a half-quire of fools-cap for $12\frac{1}{2}$ cts.; at that rate what would a ream cost?

8. What part of a quire of paper is half a score of sheets? A score of sheets?

9. What part of a ream are 5 quires? 8 quires?

10. What part of a barrel of flour are 25 lbs.? 50 lbs.? 98 lbs.? 147. lbs?

CHAPTER IX

EXERCISES IN RATIO AND PROPORTION.

LESSON I.

1. To what has 3 the relation which 1 has to 2?

ANALYSIS.—1 has to 2 the relation of $\frac{1}{2}$; 3 is $\frac{1}{2}$ of 6; therefore 3 has to 6 the relation which 1 has to 2.

NOTE.—This kind of relation is commonly called *ratio*.

2. To what has 3 the ratio which 1 has to 3?

3. To what has 4 the ratio which 1 has to 3?

4. To what has 3 the ratio which 2 has to 8?

5. To what has 4 the ratio which 2 has to 8?

6. To what has 5 the ratio which 3 has to 12?

7. To what has 5 the ratio which 3 has to 15?

8. To what has 6 the ratio which 2 has to 10?

9. To what has 6 the ratio which 3 has to 21?

10. To what has 7 the ratio which 4 has to 32?

11. To what has 8 the ratio which 5 has to 45?

12. To what has 9 the ratio which 6 has to 48?

13. To what has $\frac{1}{2}$ the ratio which 2 has to 4?

14. To what has $\frac{1}{2}$ the ratio which 2 has to 6?

15. To what has $\frac{1}{3}$ the ratio which 5 has to 10?

16. To what has $\frac{1}{4}$ the ratio which 3 has to 9?

17. To what has $\frac{2}{3}$ the ratio which 4 has to 8?

18. To what has $\frac{3}{4}$ the ratio which 6 has to 18?

19. To what has 4 the ratio which 20 has to 10?

ANALYSIS.—Since 20 has to 10 the ratio of $\frac{20}{10}$, or 2, and since 4 is twice 2; 4 has to 2 the ratio which 20 has to 10.

20. To what has 1 the ratio which 12 has to 6?

21. To what have \$8 the ratio of 5 s. to £1?
22. To what have \$8 the ratio of £1 to 5 s.?
23. To what have 12 cts. the ratio of 6 in. to 1 yd.?
24. To what have 12 cts. the ratio of 1 yd. to 6 in.?
25. To what have 16 lb. the ratio of \$1 to 25 cts.?
26. To what have 10 cwt. the ratio of \$50 to \$75?
27. To what has 1 acre the ratio of 15 pwt. to $2\frac{1}{2}$ oz.?
28. To what have 2 qt. the ratio of $2\frac{1}{2}$ A. to 10 A.?
29. To what have 6 days the ratio of \$24 to \$28?
30. To what have \$35 the ratio of 4 wk. to 4 da.?
31. To what has 1 yr. the ratio of \$10 to \$2.50?
32. To what have \$6 the ratio of 12 mo. to 4 mo.?
33. If 8 acres cost \$120, what do 5 acres cost?

COMMON ANALYSIS.—If 8 acres cost \$120, 1 acre costs $\frac{1}{8}$ of \$120, which is \$15, and 5 acres cost 5 times \$15, or \$75.

ANALYSIS BY RATIO.—Since 8 acres are $\frac{8}{5}$ of 5 acres, \$120, the cost of 8 acres, is $\frac{8}{5}$ of the cost of 5 acres. If \$120 is $\frac{8}{5}$, $\frac{1}{8}$ of \$120, or \$15, is $\frac{1}{5}$, and 5 times \$15, or \$75, are the cost of 5 acres.

34. If 10 pounds of sugar cost 95 cents, what do 8 pounds cost?

35. If 12 yards of carpet cost \$15, what would 15 yards cost?

36. If a house 30 ft. high cast a shadow 40 ft. long, how high must the tree be which, at the same time and place, would cast a shadow 64 feet long?

37. If a man's wages amount to \$75 in $2\frac{1}{2}$ months, in what time would they amount to \$225?

38. If 9 barrels of oil cost \$63, how many barrels can be bought for \$84?

39. If 6 hats cost \$16, what do 9 hats cost?

40. If a man digs a well in 20 days, by laboring 12 hours a day, how long would it take him, if he labored only 10 hours a day?

41. If a certain quantity of food lasts 7 persons 3 weeks, how long would it last 3 persons?

42. If $\frac{5}{6}$ of a yd. cost $\$ \frac{3}{4}$, what cost $\frac{3}{6}$ or $\frac{1}{2}$ of a yd.?

43. If $\frac{3}{8}$ of a yd. cost $\$ \frac{5}{8}$, what cost $\frac{7}{8}$ of a yd.?

44. If $\frac{3}{4}$ lb. of tea cost 6 shillings, what cost $\frac{1}{2}$ lb.?

LESSON II.

1. If 5 men in 10 days receive \$75 wages, how much would 15 men receive in 30 days, at the same rate?

ANALYSIS BY RATIO.—15 men would in 10 days receive $1\frac{1}{2}$, or 3 times as much as 5 men, that is 3 times \$75, or \$225, in 10 days, and in 30 days they would receive $3\frac{1}{10}$, or 3 times as much as they would in 10 days; that is 3 times \$225, or \$675.

2. If \$400 gain \$16 in 8 months, how much will \$100 gain in 12 months?

3. If \$100 gain \$6 in 12 months, how long will it take \$400 to gain \$16?

4. If the freight of 8 cwt., for 36 miles, is \$6, what would it be for 20 cwt., 18 miles, at the same rate?

5. If 4 horses eat 6 tons of hay in 9 months, how many tons would 2 horses eat in 15 months, at the same rate?

6. If keeping up 5 fires 8 months consumes 600 bu. of coal, how much coal, at the same rate, would be consumed by keeping up 3 fires 9 months?

7. If a landlady receives \$60 for the boarding of 3 men 5 weeks, how much, at the same price, would she receive for the boarding of 7 men 4 weeks?

8. If the 6 cent loaf weighs 12 oz. when flour is \$6 a barrel, how much should the 8 cent loaf weigh when flour is \$4 a barrel?

9. If 8 men mow 20 A. in 2 da. of 12 hr. each, how many acres would 10 men mow in 3 days of 10 hr. each?

10. Two men bought 21 acres of land together, and shared it in parts which were as 3 to 4, as each had furnished pay for it; how much land had each?

ANALYSIS.—If the *shares* were as 3 units to 4 units, the *whole* would be $3 + 4$, or 7 units, and one man would receive $\frac{3}{7}$ of 21 acres, or 9 acres, and the other $\frac{4}{7}$ of 21 acres, or 12 acres.

11. Two boys bought an orange for 3 cents, one furnishing 1 cent, the other 2 cents; if they share the orange proportionally, what part of the orange has each?

12. Divide 48 into parts which are as 5 to 7.

13. Divide 72 into parts which shall be as 3 to 5 ; as 4 to 5 ; as 5 to 7 ; as 7 to 11 ; as 11 to 13 ; as 15 to 21.

14. Two men ventured in a speculation, one furnishing \$200, the other \$300 ; when they divide the profits \$1000, how much does each receive ?

15. Two men paid \$40 for the use of a pasture 5 months ; one put in 3 cows, the other 5 cows ; how much should each pay ?

16. A bankrupt, in settling with his creditors, paid 60 cents on the dollar ; how much did B receive, to whom he owed \$500 ?

17. A owes B \$200, C \$300, and D \$400 ; he gives up his property worth \$450 ; how much should B, C, and D receive ?

18. A and B, partners, divided a profit of \$800 ; A had furnished \$50 for 6 months, and B \$100 for 5 months ; how much should each receive ?

19. A, B, and C, hired a pasture for \$80 ; A put in 1 cow 5 months, B 2 cows 3 months, and C 3 cows 3 months ; how much should each pay ?

20. Divide \$96 so that the parts shall be to each other as the numbers 1, 2, and 3.

21. Divide \$100 so that the parts shall be to each other as the numbers 1, 2, 3, and 4.

22. D, E, and F, on dissolving partnership, found that the firm was insolvent \$24000 ; D had furnished \$3000, E \$4000, and F \$5000 ; how much of the debt of the firm should each pay ?

23. Four men in partnership divided the stock into 40 shares ; A owned 6 shares, B 8, C 12, and D 14 ; how should they divide their profits, or losses ?

24. An insurance company's stock was divided into 500 shares ; on paying for their losses one year, amounting to 1500 dollars, how much had A to pay, who owned 10 shares ?

25. The stock of a bank is divided into 1000 shares ; when they make a dividend of \$6000, how much does B receive, who owns 24 shares ?

CHAPTER X.

EXERCISES IN PERCENTAGE.

LESSON I.

1. WHAT is meant by 5 *per cent.* of a quantity?

ANS. Five per cent. of a quantity is 5 hundredths of that quantity; and, in general, any per cent. of a quantity is so many hundredths of it.

2. What is the meaning of *per cent.*?

ANS. It is an expression derived from the Latin, and means *by the hundred*.

NOTE.—For brevity the symbol % is often used for the words *per cent.* It is also plain that any per cent. can be written fractionally or decimally; thus 6 per cent. may be written 6%, or $\frac{6}{100}$, or .06.

3. How much of a quantity is 1% of it?

ANS. It is 1 hundredth of it.

4. How much of a quantity is $\frac{1}{2}$ % of it?

ANS. It is $\frac{1}{2}$ of 1 hundredth, or $\frac{1}{200}$ of it.

5. How much is 200% of a quantity?

ANS. Since 100% is $\frac{100}{100}$, or the whole of it, 200% of a quantity must be twice that quantity; 150% of it must be $\frac{150}{100}$ or $1\frac{1}{2}$ times that quantity.

6. What is 1% of \$100? \$200? 100 cts.?

7. What is 2% of 100 bushels of wheat?

8. What is 3% of 200 tons of iron?

9. What is 4% of 600 men?

10. What is 5% of 300 acres of land?

11. What is 6% of 500 barrels of apples?

12. What is 10% of \$700? \$800? \$900?

13. What is 2% of \$50?

FIRST ANALYSIS.—1% or $\frac{1}{100}$ of \$50 is $\frac{50}{100}$ or $\frac{1}{2}$ of \$1, and 2% must be twice as much, or \$1. Therefore 2% of \$50 is \$1.

SECOND ANALYSIS.—2% or $\frac{2}{100}$ of \$50 is 1% or $\frac{1}{100}$ of twice \$50, that is, $\frac{1}{100}$ of \$100, which is \$1. Therefore 2% of \$50 is \$1.

NOTE.—Decimally, this would be $\$50 \times .02 = \1.00 .

14. What is 2% of \$75? Of \$125?

15. What is 7% of \$7? Of \$70? Of \$35?

16. What is 4% of \$15? Of \$47? Of \$2 $\frac{1}{2}$?

17. What is 5% of \$20? Of \$63? Of \$8 $\frac{2}{3}$?

18. What is 6% of 50 sheep? Of 25 acres?

19. What is 3% of 133 $\frac{1}{3}$ lb. of flour?

20. What is 8% of 75 bushels of wheat?

21. What is 10% of 125 yards of cloth?

22. What is $\frac{1}{2}$ % of \$100?

ANALYSIS.—1% of \$100 is $\frac{1}{100}$ of \$100, or \$1, and $\frac{1}{2}$ %, which is $\frac{1}{2}$ of 1%, must be $\frac{1}{2}$ of \$1, or 50 cts. Therefore $\frac{1}{2}$ % of \$100 is 50 cts.

23. What is $\frac{1}{2}$ % of \$20? \$30? \$40? \$50?

24. What is $\frac{3}{4}$ % of \$60? \$80? \$120? \$200?

25. What is $\frac{2}{5}$ % of \$10? \$70? \$90? \$150?

26. What is 1 $\frac{1}{8}$ % of \$48? \$64? \$88? \$32?

27. What is 2 $\frac{3}{10}$ % of \$30? \$50? \$80? \$90?

28. What part of a quantity is 2% of it?

ANALYSIS.—2% of a quantity is $\frac{2}{100}$, or $\frac{1}{50}$ of it.

29. What part of a quantity is 2 $\frac{1}{2}$ per cent. of it?

ANAL.—2 $\frac{1}{2}$ % of a quantity is 2 $\frac{1}{2}$ hundredths, or $\frac{5}{200}$ or $\frac{1}{40}$ of it.

30. What part of a quantity is 4% of it?

31. What part of a quantity is 5% of it?

32. What part of a quantity is 6% of it?

33. What part of a quantity is 6 $\frac{1}{4}$ % of it?

34. What part of a quantity is 12 $\frac{1}{2}$ % of it? 37 $\frac{1}{2}$ %?

35. What part of a quantity is 10% of it?

36. What part of a quantity is 8% of it?

37. What part of a quantity is 7 $\frac{1}{2}$ % of it? 15%?

38. What part of a quantity is 16 $\frac{2}{3}$ % of it? 33 $\frac{1}{3}$ %?

39. What part of a quantity is 20% of it? 40%?

40. What part of a quantity is 25% of it? 50%?

41. What part of a quantity is 75% of it?

LESSON II.

1. What per cent. of a quantity is $\frac{3}{10}$ of it?

ANALYSIS.—Since $\frac{100}{100}$ or 100% make a quantity, $\frac{3}{10}$ of that quantity must be $\frac{3}{10}$ of 100%, or 30% of it. Therefore $\frac{3}{10}$ of a quantity is 30% of it.

2. What per cent. of a quantity is $\frac{2}{5}$ of it?

3. What per cent. of a quantity is $\frac{1}{4}$ of it?

4. What per cent. of a quantity is $\frac{1}{3}$ of it?

5. A farmer, having gathered 80 bushels of potatoes, lost $\frac{3}{8}$ of them; what per cent. did he save?

6. An army lost in battle $\frac{1}{10}$ of its men, and $\frac{1}{5}$ by desertion; what per cent. was it reduced?

7. A man having 30 sheep, sold 6 of them; what per cent. of them did he sell?

ANALYSIS.—He sold $\frac{6}{30}$ or $\frac{1}{5}$ of them; $\frac{1}{5}$ of 100% is 20%. Therefore he sold 20% of them.

8. From a barrel of wine, containing 40 gallons, were drawn 5 gallons; what per cent. was drawn from it, and what per cent. remained?

9. From a piece of cloth containing 45 yds., were cut 27 yds.; what per cent. of the piece remained?

10. A man gave \$50 in charities; to one he gave \$3, and to others respectively \$5, \$6, \$8, \$10, and \$18; what per cent. of the whole was each gift?

11. What per cent. of 25 is 7? 9? 11? $12\frac{1}{2}$? 16? 17? 18? 19? 20? 21? 22? 23? 24?

12. What per cent. of 75 is 15? 9? 6? 12? 18? 21? 24? 27? 30? 33? 36? 39?

13. What per cent. of $17\frac{1}{2}$ is $3\frac{1}{2}$? 7? $10\frac{1}{2}$?

14. A man invested \$150, and from it realized a profit of \$60; what per cent. was his profit?

15. A man invested \$225, and realized only \$150; what per cent. did he lose?

16. What per cent. of $37\frac{1}{2}$ cts. is $6\frac{1}{4}$ cts.?

17. For a house which cost 25 hundred dollars, the owner asks \$275 rent; what per cent. upon the cost of the property is the rent?

18. My banker collected for me a debt of \$120, and charged 5% commission; how much did he retain, and how much did he pay me?

19. A tax-collector collected \$500 taxes, and retained 4% for fees; how much did he pay over?

20. An attorney collected \$350 for his client, and retained 10% for fees; what did the client receive?

21. A broker sold for a man \$960 worth of railroad stock, and charged $\frac{1}{2}$ % brokerage; how much did the owner of the stock realize?

22. A man, arriving at Pittsburgh from St. Louis, exchanged \$350 St. Louis money for Pittsburgh money, paying $1\frac{1}{2}$ % exchange; how much did he receive in Pittsburgh money?

23. An auctioneer sold goods for a man to the amount of \$475, and charged 8% for his trouble; how much did the man receive for his goods?

24. An agent sold \$750 worth of flour for a dealer, and charged 4% for selling, and 2% for storage; how much remained to remit to the dealer?

25. A bankrupt, on settling his affairs, could only pay $62\frac{1}{2}$ % of his debts; how much did A receive, whose claim was \$336?

26. A merchant imported goods invoiced as worth \$2500, and paid on them 24% duties at the custom-house; what did the goods cost him?

27. The firm of Case & Co. imported goods invoiced at \$1800, but being damaged, the collector valued them $33\frac{1}{3}$ % less, and on this valuation collected 15% duties; what did the goods cost Case & Co.?

28. In a certain town the taxes for public purposes were $2\frac{7}{10}$ %; how much were Mr. Smith's taxes, whose property was valued at \$1000?

29. H. Mann bought a share in an oil-well for \$50, and sold it for 350% of its cost; how much did he get for it?

30. What is 175% of \$80? $237\frac{1}{2}$ % of \$24?

31. If an article is sold for 500% of its cost, what per cent. is the gain?

LESSON III.

1. A man got his house insured for \$800, paying a premium of $2\frac{3}{4}\%$; what did he pay for insurance?

NOTE.—The amount paid for insurance is called the *premium*.

2. If a mill is insured for \$20000, at 5% , what is the amount of premium paid the insurance company?

3. If a barn and its contents are insured for \$600, at $3\frac{1}{2}\%$, what premium is paid for the insurance?

4. If a shop and its contents are insured for \$1500, at $4\frac{2}{5}\%$, what is the premium?

5. If a steamboat is insured for \$10000, at $6\frac{1}{4}\%$, what does the insurance cost?

6. If a ship is insured for \$50000, at 10% , what does her insurance cost?

7. If a company insures goods valued at \$500, at 2% premium, what is the loss of the company, if the goods are destroyed?

ANALYSIS.—The company receive 2% of \$500, which is \$10, and they pay \$500; therefore their loss is \$500 — \$10, or \$490.

8. Goods insured at \$800, 3% premium, were lost; what was the loss of the insurance company?

9. A man got his house insured for \$700, at $2\frac{1}{2}\%$; when the house was burned, what was the loss of the insurance company? If the house cost \$1400, what was the total loss of its owner?

10. A man had paid annually for 5 years a premium of $3\frac{1}{4}\%$ on his store valued at \$1600; when it was damaged to the extent of \$520, the company paid the damage; what was their loss? The owner's loss?

11. If a person insures his life for \$3000, at $2\frac{1}{5}\%$ annual premium, and dies at the end of 10 years, what do his heirs receive more than he paid?

12. If he dies at the end of 20 years, how much more do his heirs receive than he paid?

13. If he dies at the end of 30 years, how much more do his heirs receive than he paid?

14. A sends B \$420 to expend it in the purchase of flour so that the flour and B's commission at 5% would take the whole \$420; how much did B expend?

ANALYSIS.—If what B expends is $\frac{100}{105}$, and B's commission on it is $\frac{5}{105}$, then the cost of the flour and commission are $\frac{100}{105}$, or $\frac{20}{21}$, of the sum expended. If \$420 is $\frac{20}{21}$ of the sum expended, $\frac{21}{20}$ of \$420, or \$20, is $\frac{1}{20}$ of that sum, and 20 times \$20, or \$400, is that sum.

15. If \$520 includes both the expenditure and the commission on it at 4%, what is the expenditure?

16. If \$682 includes expenditure and its commission at 10%, what is the expenditure?

17. If \$594 includes expenditure and its commission at 8%, what is the expenditure?

18. A broker received \$820 with orders to purchase bank-stock; how many shares could he buy at \$50 each, his commission $2\frac{1}{2}\%$ being taken out?

19. An agent received \$306 with orders to buy wheat; how many bushels, at \$1 a bushel, could he buy, retaining a commission at 2%?

20. A's shop cost \$900; for how much must he get it insured at 10% so as to suffer no loss in case the shop is burned?

ANALYSIS.—If A suffers no loss, he must receive from the insurance company \$900, the cost of the shop, and whatever premium he paid them. Therefore he must insure for the sum of cost and premium. Since the premium is 10% of the amount insured, the cost \$900 must be 100%—10%, or 90% of the amount insured. If \$900 is 90%, $\frac{1}{90}$ of \$900, or \$10, is 1%, and 100 times \$10, or \$1000, must be 100% of the amount insured, or the whole of it.

21. For how much must a house costing \$1470 be insured at 2%, to cause the insured no loss, in case it is destroyed by fire?

22. How much money must a tax-collector collect, that, after deducting his commission of 5%, he may pay the treasurer \$456?

23. How much tax must a town assess upon itself, that, after allowing 10% for uncollectable taxes and the expenses of collection, it may realize \$7200?

LESSON IV.

1. A man bought a horse for \$80, and sold him at 10 % gain; for how much did he sell the horse?

FIRST ANALYSIS.—He sold the horse at $\frac{11}{10}$ or $\frac{11}{10}$ of what he gave; $\frac{11}{10}$ of \$80 is \$88; therefore he sold the horse for \$88.

SECOND ANALYSIS.—He gained 10 % or $\frac{1}{10}$ of \$80, which is \$8; \$80 + \$8 are \$88. Therefore he sold the horse for \$88.

2. For how much must goods, which cost \$60, be sold to gain 20 % ? 10 % ? 5 % ?

3. Bought a lot of flour for \$150, and sold it at 8 % gain; how much did I get for the flour?

4. Bought coffee for \$72, and retailed it at an advance of $12\frac{1}{2}$ % ; how much did I gain?

5. A speculator bought a lot of land for \$200, and sold it at a gain of 50 % ; how much did he gain?

6. A man gave \$300 for some bank-stock, and sold it at 25 % advance; what did he get for the stock?

7. If you buy eggs at 9 cts. a dozen, at what price must you sell them to gain $33\frac{1}{3}$ % ?

8. A merchant buys silk at 90 cts. a yard; at how much per yard must he sell it, to gain 40 % ?

9. A grocer buys sugar at 10 cts. per lb. ; at what price per pound must he sell it, to gain 20 % ?

10. A shoe-dealer buys shoes at \$1.50 per pair; how must he sell them, to gain 30 % ?

11. A miller bought wheat for \$600, but realized a loss of $16\frac{2}{3}$ % on its sale; what did he get for it?

ANALYSIS.—He lost $16\frac{2}{3}$ %, or $\frac{1}{6}$ of \$600, which is \$100; therefore he got \$600 less \$100, or \$500 for it.

12. A fruit-dealer bought apples for \$250, but sold them at 20 % loss; how much did he get for them?

13. A man gave \$36 for a cow, but sold her at 25 % loss; how much did he get for her?

14. A man began business with 10 thousand dollars, but left it with a loss of 40 % of his capital; with how much did he leave the business?

15. A certain town had, at one census, a population of 20 thousand; but at the next census, it had diminished 10 %; what was its population then?

16. A merchant, on receiving a new lot of goods, marked his old goods at $12\frac{1}{2}\%$ loss; what did he mark gingham which cost him 16 cents per yard?

17. If the cost of any thing is \$16 and its selling price is \$20, what is the gain per cent.?

ANALYSIS.—The gain is \$20 — \$16, or \$4; and \$4 is $\frac{1}{4}$ of the cost \$16; if \$16 is $\frac{100}{100}$ or 100 % of the cost, $\frac{1}{4}$ of \$16 is $\frac{1}{4}$ of 100 %, or 25 % of the cost. Therefore the gain is 25 %.

18. Cost \$25; sale at \$30; required gain per cent.

19. Cost \$40; sale at \$45; required gain per cent.

20. Cost \$10; sale at \$20; required gain per cent.

21. Cost \$8; sale at \$18; required gain per cent.

22. Cost \$12; sale at \$30; required gain per cent.

23. Cost \$4; sale at \$11; required gain per cent.

24. Cost \$6; sale at \$20; required gain per cent.

25. Cost \$16; sale at \$50; required gain per cent.

26. Cost 5 cts.; sale 4 cts.; required loss per cent.

27. Cost $12\frac{1}{2}$ cts.; sale 12 cts.; what loss per cent.?

28. Cost $12\frac{1}{2}$ cts.; sale 13 cts.; what gain per cent.?

29. Cost 30; sale 27; what is the loss per cent.?

30. Cost 30; sale 33; what is the gain per cent.?

31. Cost 200; sale 201; what is the gain per cent.?

32. Cost 50; sale $49\frac{3}{4}$; what is the loss per cent.?

33. If I begin business with \$1000, and close it with \$1500, what per cent. has my capital increased?

34. If I begin with \$500, and close with \$2500?

35. If I begin with \$1200, and close with \$800, how much per cent. has my capital decreased?

36. If a man receives \$800 salary, and lives on \$700, what per cent. of his salary does he save?

37. If a boy buys oranges at 20 cts. a dozen, and sells them at 3 cents apiece, what per cent. profit has he?

38. If a news-boy buys papers at the rate of 10 for 6 cents, and sells them at 1 cent apiece, what per cent. does he make. and how much on 50 papers?

LESSON V.

1. A merchant, some of whose customers "beat him down" in his prices, determined to humor them by fixing prices so high that he could safely reduce them; how should he mark silks which cost \$1 per yard, so as to reduce his price 10 %, and still make 35 % ?

ANALYSIS.—The silk costs \$1, and sells at 35 % gain; therefore it sells at $\$1 + 35$ cents, or \$1.35. If \$1.35 is 10 % less than marked price, it is 90 % or $\frac{9}{10}$ of marked price, and $\frac{1}{9}$ of \$1.35, which is 15 cents, is $\frac{1}{10}$ of marked price, and 10 times 15 cents, which is 150 cents, or \$1.50, must be the marked price.

2. How should he mark gingham which cost 15 cents, so as to fall 10 %, and still make 20 % ?

3. How should he mark muslins which cost 8 cts., so as to fall 10 %, and still make $12\frac{1}{2}$ % ?

4. How should he mark velvet which cost 25 cts., so as to fall 10 %, and still make 44 % ?

5. How should he mark merino which cost 50 cts., so as to fall 5 %, and still make 14 % ?

6. How should he mark ribbon which cost 12 cts., so as to fall $16\frac{2}{3}$ %, and still make 25 % ?

7. How should goods which cost 60 cts. be marked that the seller may fall $6\frac{1}{4}$ %, and still make 25 % ?

8. How should goods which cost \$5 be marked, that the seller may fall $12\frac{1}{2}$ %, and still make 40 % ?

9. A grocer sells sugar at 12 cts. per lb., thereby gaining 20 %; what did the sugar cost him ?

ANALYSIS.—If he gains 20 %, or $\frac{1}{5}$ of the cost, 12 cts., his selling price, is $\frac{6}{5}$ of the cost, and $\frac{1}{6}$ of 12 cts., which is 2 cts., must be $\frac{1}{5}$ of cost; therefore cost must be 5 times 2 cts., which is 10 cts.

10. A grocer sells coffee at 15 cts. per lb., thereby gaining 20 %; what did the coffee cost him ?

11. He sells tea at 75 cts. per pound, thereby gaining 50 %; what did the tea cost him ?

12. A merchant sells cloth at 64 cts., thereby gaining $33\frac{1}{3}$ %; what did the cloth cost ?

13. A man sold his house for \$1200, which was 25% loss; what did the house cost him?

14. A farmer sold a horse for \$80, which was 20% less than he gave for him; what did he give for him?

15. If by selling paper at 10 cts. per quire, I lose $33\frac{1}{3}\%$, what did the paper cost me?

16. A \$5 bank-note was worth only \$4.75; what per cent. below its nominal value was its real value?

17. A \$10 bank-note was worth \$9.90; what per cent. below its nominal value was its real value?

18. A man exchanging bank-bills for gold, had to give \$51 in bills for \$50 in gold; at what per cent. premium was the gold?

19. \$100.50 in silver was paid for \$100 in gold; at what per cent. premium was the gold?

20. If I receive $\frac{5}{4}$ of any investment in its returns, what per cent. gain do I realize? $\frac{3}{2}$? $\frac{4}{3}$? $\frac{5}{3}$? $\frac{5}{2}$? $\frac{6}{5}$? $\frac{7}{5}$? $\frac{8}{5}$? $\frac{9}{5}$? $\frac{10}{5}$? $\frac{11}{10}$? $\frac{13}{10}$? $\frac{9}{8}$? $\frac{11}{8}$?

21. If I only get returns of $\frac{2}{3}$ of my investment, what per cent. loss do I suffer? $\frac{3}{4}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{2}{5}$? $\frac{3}{8}$? $\frac{1}{2}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{3}{10}$? $\frac{7}{10}$? $\frac{9}{10}$?

22. If an attorney collects \$680 for me, and charges 5% commission, how much does he deliver to me?

23. If, by selling sugar at 10 cts. per lb., a merchant gains 25%, what per cent. would he gain if he sold it at 11 cts. per lb? At 12 cts. per lb.?

24. A merchant buys sugar at the rate of 10 pounds for \$1, and sells it at the rate of 9 pounds for \$1; what per cent. profit does he make?

25. If a boy buys apples at the rate of 3 for 2 cents, and sells them at the rate of 2 for 3 cents, what per cent. profit does he make?

26. If a town allows its tax collectors 5%, and allows \$100 for uncollected taxes, how much tax should it assess to realize \$1900?

27. A man sent \$1545 to his agent to expend for wheat; if the agent charges 3% commission, how much should be expended as desired?

CHAPTER XI.

EXERCISES IN INTEREST AND DISCOUNT.

LESSON I.

1. If I borrow \$100 for a year, and at the end of the year pay the lender \$106, what per cent. do I pay the lender for the use of his money?

ANS. 6%, because \$6 is $\frac{6}{100}$ of \$100.

2. What is the \$6 called in reference to the \$100?

ANS. It is called the *interest* of the \$100.

3. What is interest?

ANS. Interest is a compensation paid for the use of money.

4. What is the sum of money called on which interest is paid?

ANS. It is called the *principal*.

5. What is the sum of principal and interest called?

ANS. It is called the *amount*.

NOTE.—Interest is usually reckoned either by the year or month. When no time is mentioned, interest is understood to be reckoned *per annum*, or by the year. Most States fix by law the rate of interest, and the taking of a higher rate of interest than the legal rate is called *usury*.

6. At 5% per annum, what part of the principal equals the yearly interest?

ANS. $\frac{5}{100}$ or $\frac{1}{20}$ of the principal.

7. At 4% per annum, what part of the principal equals the annual interest? At 2%? At 6%? At 8%? At 10%? At 12%? At 3%? At 7%? At 9%?

8. At 1% a month, what part of the principal equals the monthly interest? At 2% a month?

9. What is the yearly interest of \$60 at 5%?

FIRST ANALYSIS.—At 5% a year, the yearly interest is $\frac{1}{20}$ of the principal; $\frac{1}{20}$ of \$60 is \$3; therefore at 5% the yearly interest of \$60 is \$3.

SECOND ANALYSIS.—Since 5% interest of \$1 or 100 cts. is 5 cts., 5% interest of \$60 is 60 times 5 cts., which is 300 cts., or \$3.

NOTE.—Decimally, this process would be $\$60 \times .05 = \3.00 .

10. What is the yearly interest of \$180 at 5%? Of \$120? Of \$400? Of \$700? Of \$940? Of \$1000?

11. What is the annual interest of \$75 at 4%? Of \$125? Of \$250? Of \$600? Of \$1000?

12. What is the annual interest of \$50 at 6%? Of 50 cents? Of \$1.50? Of \$150? Of \$1000?

13. What is the annual interest of \$100 at 7%? Of \$1? Of \$50? Of 50 cts.? Of \$25? Of \$1000?

14. What is the annual interest of \$75 at 8%? Of 75 cts.? Of \$225? Of \$50.25? Of \$500? Of \$1000?

15. What is the annual interest of \$90 at 10%? Of \$45? Of \$750? Of \$125.50? Of \$1000?

16. What is the annual interest of \$100 at 9%? Of \$50? Of \$25? Of \$500? Of \$70? Of \$1000?

17. What is the annual interest of \$100 at 11%? Of \$50? Of \$25? Of \$600? Of \$7.20? Of \$1000?

18. What is the annual interest of \$150 at 12%? Of \$75? Of 75 cts.? Of \$75.75? Of \$500? Of \$1000?

19. What is the annual interest of \$10 at 15%? Of \$40? Of \$160? Of \$300? Of \$1000?

20. What is the annual interest of \$80 at $12\frac{1}{2}\%$? Of \$240? Of 64 cts.? Of \$24.80? Of \$1000?

21. What is the annual interest of \$20 at 20%? Of \$25? Of \$180? Of 50 cts.? Of \$5.50? Of \$1000?

22. What is the monthly interest of \$100 at 3% a month? Of \$50? Of \$200? Of \$1000?

23. What is the monthly interest of \$50 at 2% a month? Of \$100? Of \$25? Of \$150? Of \$1000?

24. What is the monthly interest of \$100 at 1% a month? At $1\frac{1}{2}\%$ a month? At $1\frac{1}{4}\%$ a month?

LESSON II.

1. What is the interest of \$40 for 4 years, at 5%?

FIRST ANALYSIS.—For 1 year it is 5%, and for 4 years it is 4 times 5%, which is 20%, or $\frac{1}{5}$, of \$40, which is \$8.

OR, For 1 year it is 5%, or $\frac{1}{20}$, and for 4 years it is 4 times $\frac{1}{20}$, which is $\frac{4}{20}$, or $\frac{1}{5}$, of \$40, which is \$8.

SECOND ANALYSIS.—For 1 year it is 5% of \$40, which is \$2, and for 4 years it is 4 times \$2, or \$8.

NOTE.—Decimally, these processes would be $.05 \times 4 = .20$; $\$40 \times .20 = \8.00 .

OR, $\$40 \times .05 = \2 . $\$2 \times 4 = \8 .

2. What is the interest of \$60 for 2 yrs., at 5%? For 3 yrs.? 4 yrs.? $4\frac{1}{2}$ yrs.? 5 yrs.? $6\frac{1}{2}$ yrs.?

3. What is the interest of \$50 for 2 yrs., at 4%? For 3 yrs.? $4\frac{1}{2}$ yrs.? 5 yrs.? $6\frac{1}{2}$ yrs.? 10 yrs.?

4. What is the interest of \$150 for 2 yrs., at 6%? For $3\frac{1}{2}$ yrs.? $5\frac{1}{2}$ yrs.? 6 yrs.? 8 yrs.? 10 yrs.?

5. What is the interest of \$175 for 2 yrs., at 7%? For 4 yrs.? $6\frac{1}{2}$ yrs.? 8 yrs.? 10 yrs.?

6. What is the interest of \$80 for $2\frac{1}{2}$ yrs., at 8%?

7. Borrowed \$135 at 10%; what interest is due at the end of 2 yrs.? 3 yrs.? 4 yrs.? 5 yrs.? 10 yrs.?

8. Loaned \$50 at 9%; what interest is due me at the end of $1\frac{1}{2}$ yrs.? 2 yrs.? 10 yrs.?

9. Bought a \$1000 3% bond; if the interest is paid, how much would I receive in 5 yrs.?

10. A merchant borrowed \$400 at 2% a month; what interest must he pay in 3 months? In 5 months?

11. A broker loaned \$150 at $1\frac{1}{2}$ % a month; what interest was due him in 2 months?

12. What is the interest of \$60 for 4 months, at $1\frac{1}{4}$ % a month?

13. A man took 5% a month from a borrower; at that rate, what part of the principal would 1 year's interest be?

14. At 20% a year, what is the interest of \$55 for 2 yrs.? 3 yrs.? 4 yrs.? 5 yrs.? 10 yrs.?

15. What is the interest of \$25 for 4 yrs., at 25%?

16. At 6%, what is the interest of \$100 for 2 years and 8 months?

FIRST ANALYSIS.—Since 12 months are a year, 8 months are $\frac{8}{12}$ or $\frac{2}{3}$ of a year, and 2 yrs. 8 mo. are $2\frac{2}{3}$ years. At 6%, the interest for 1 yr. is 6%, for 2 yr. twice 6%, or 12%, and for $\frac{2}{3}$ of a year $\frac{2}{3}$ of 6%, or 4%; therefore for $2\frac{2}{3}$ yrs. it would be 12% + 4% = 16%, or $\frac{4}{25}$ of \$100 = \$16.

SECOND ANALYSIS.—For 1 year the interest is 6% of \$100, which is \$6, for 2 years it is 2 times \$6, which is \$12, and for 8 months, or $\frac{2}{3}$ of a year, it is $\frac{2}{3}$ of \$6, which is \$4. Therefore for 2 yrs. and 8 months it is \$12 + \$4, or \$16.

NOTE.—Decimally, this would be $\$100 \times .06 = \6.00 , $2\frac{2}{3} \times \$6 = \16 .

17. At 6%, what is the interest of \$50 for 3 yr. 6 m.?

18. At 6%, what is the interest of \$75 for 4 yr. 3 m.?

19. At 6%, what is the interest of \$150 for 5 yr. 4 m.?

20. At 6%, what is the interest of \$250 for 2 yr. 9 m.?

21. At 5%, what is the interest of \$40 for 6 yr. 6 m.?

22. At 5%, what is the interest of \$60 for 4 yr. 4 m.?

23. At 5%, what is the interest of \$80 for 3 yr. 3 m.?

24. At 5%, what is the interest of \$120 for 2 yr. 10 m.?

25. At 5%, what is the interest of \$240 for 5 yr. 11 m.?

26. At 7%, what is the interest of \$50 for 8 yr. 4 m.?

27. At 7%, what is the interest of \$10 for 3 yr. 3 m.?

28. At 8%, what is the interest of \$25 for 2 yr. 6 m.?

29. At 8%, what is the interest of \$300 for 2 yr. 7 m.?

30. At 8%, what is the interest of \$75 for 5 yr. 2 m.?

31. At 8%, what is the interest of \$60 for 2 yr. 1 m.?

32. At 9%, what is the interest of \$200 for 3 yr. 4 m.?

33. At 9%, what is the interest of \$400 for 1 yr. 5 m.?

34. At 9%, what is the interest of \$80 for 6 yr. 8 m.?

35. At 10%, what is the interest of \$120 for 2 yr. 7 m.?

36. At 10%, what is the interest of \$36 for 5 months?

37. At 12%, what is the interest of \$25 for 6 months?

38. At $12\frac{1}{2}\%$, what is the interest of \$80 for 9 months?

39. At 15%, what is the interest of \$40 for 10 months?

40. At 3%, what is the interest of \$500 for 1 month?

41. At 4%, what is the interest of \$150 for $\frac{1}{2}$ month?

42. At 20%, what is the interest of \$90 for 7 months?

43. At 25%, what is the interest of \$48 for 11 months?

LESSON III.

In computing interest for portions of a month, it is customary to reckon 30 days as a month. Although 12 such months make only 360 days, which is not quite a year, yet the error resulting from this reckoning is small. The *exact* interest for days would be as many 365ths of a year's interest as there are days.

1. At 6%, what is the interest of \$200 for 1 year, 2 months and 10 days?

FIRST ANALYSIS.—For 1 year it is 6%, for 2 months it is $\frac{2}{12}$, which is $\frac{1}{6}$ of 6%, or 1%. Since 10 days are $\frac{10}{30}$, or $\frac{1}{3}$ of 1 month, or $\frac{1}{6}$ of 2 months, the interest for 10 days is $\frac{1}{6}$ of 1%. Therefore the interest for the whole time is $6\% + 1\% + \frac{1}{6}\% = 7\frac{1}{6}\%$ of \$200 = \$14 $\frac{1}{3}$.

SECOND ANALYSIS.—For 1 year it is 6% of \$200, which is \$12; for 2 months it is $\frac{2}{12}$, or $\frac{1}{6}$ of \$12, which is \$2, and since 60 days are 2 months, for 10 days it is $\frac{10}{60}$ or $\frac{1}{6}$ of \$2, which is $\frac{2}{6}$, or $\frac{1}{3}$. Therefore for the whole time it is $\$12 + \$2 + \frac{1}{3} = \$14\frac{1}{3}$.

2. At 6%, what is the interest of \$100 for 2 years, 6 months and 15 days?

3. At 5%, what is the interest of \$60 for 4 years, 4 months and 20 days?

4. At 6 $\frac{1}{4}$ %, what is the interest of \$64 for 3 years, 3 months and 6 days?

5. At 7%, what is the interest of \$600 for 2 years, 1 month and 18 days?

6. At 8%, what is the interest of \$125 for 5 years, 9 months and 18 days?

7. At 9%, what is the interest of \$30 for 1 year, 8 months and 20 days?

8. At 10%, what is the interest of \$240 for 6 years, 5 months and 25 days?

9. At 12%, what is the interest of \$100 for 6 months and 12 days?

10. At 20%, what is the interest of \$120 for 11 months and 21 days?

11. At 25%, what is the interest of \$144 for 2 months and 9 days? For 3 months and 10 days?

NOTE.—At 6%, the interest of \$1 for a year is 6 cents; for 1 month it is $\frac{1}{12}$ of 6 cts., or $\frac{1}{2}$ ct., or 5 mills. If the interest of \$1 for 30 days is 5 mills, for $\frac{1}{3}$ of 30 days, or 6 days, it is 1 mill, and for one day it is $\frac{1}{6}$ of a mill. Hence, interest at 6% is often computed by calling $\frac{1}{6}$ of the days mills and multiplying this result by the principal.

12. At 6%, what is the interest of \$100 from April 1st to July 6th?

ANALYSIS.—After April 1st there are 29 days in April, 31 in May, 30 in June, and 6 in July; in all 96 days: $\frac{1}{6}$ of 96 days is 16 days, which, called mills and multiplied by the principal 100, would be 1600 mills, equal to \$1.60.

13. At 6%, what is the interest of \$100 from March 1st to April 30th? To May 12th? To June 17th?

14. At 6%, what is the interest of \$50 from May 7th to August 5th?

15. At 6%, what is the interest of \$80 from June 12th to October 10th?

16. At 6%, what is the interest of \$40 from July 4th to September 2d?

17. At 6%, what is the interest of \$150 from August 19th to October 30th?

18. At 6%, what is the interest of \$90 from August 31st to November 29th?

19. At 6%, what is the interest of \$200 from September 6th to November 11th?

20. At 6%, what is the interest of \$120 from October 1st to December 30th?

21. At 6%, what is the interest of \$75 from September 3d to January 1st?

22. At 6%, what is the interest of \$125 from December 2d to January 31st?

23. At 6%, what is the interest of \$500 from January 1st to March 31st in leap-year?

24. At 6%, what is the interest of \$300 from January 7th to April 1st in a common year? From January 7th to April 30th in a leap-year?

LESSON IV.

1. At 7%, what is the interest of \$100 for 90 days?

ANALYSIS.—At 6%, it is 100 times $\frac{1}{6}$ of 90 mills, that is, 100 times 15 mills, equal to \$1.50. At 7%, it is $\frac{7}{6}$ of \$1.50, or \$1.75.

2. At 7%, what is the interest of \$50 for 180 days?

3. At 7%, what is the interest of \$80 for 72 days?

4. At 7%, what is the interest of \$60 for 120 days?

5. At 5%, what is the interest of \$100 for 72 days?

6. At 5%, what is the interest of \$90 for 60 days?

7. At 5%, what is the interest of \$45 for 48 days?

8. At 5%, what is the interest of \$120 for 30 days?

9. At 8%, what is the interest of \$72 for 42 days?

10. At 8%, what is the interest of \$48 for 66 days?

11. At 8%, what is the interest of \$40 for 144 days?

12. At 9%, what is the interest of \$50 for 120 days?

13. At 9%, what is the interest of \$180 for 30 days?

14. At 4%, what is the interest of \$150 for 36 days?

15. At 4%, what is the interest of \$100 for 72 days?

16. At 10%, what is the interest of \$50 for 144 days?

17. At 10%, what is the interest of \$72 for 60 days?

18. At 11%, what is the interest of \$60 for 60 days?

19. At 3%, what is the interest of \$60 for 60 days?

20. At 6%, what will be the amount of \$100 in 2 years, 6 months and 15 days?

FIRST ANALYSIS.—The interest for 2 years is 12%, for 6 m. 3%, and for 15 da. $\frac{1}{4}\%$; therefore for the whole time it is $15\frac{1}{4}\%$ of the principal, or \$15 $\frac{1}{4}$, which with the principal \$100 makes the amount \$115 $\frac{1}{4}$.

SECOND ANALYSIS.—Since the principal is $\frac{100}{100}$ of itself, and the interest $\frac{15\frac{1}{4}}{100}$ of the principal, the amount is $\frac{115\frac{1}{4}}{100}$ of the principal: $\frac{1}{100}$ of the principal is \$1, and $\frac{115\frac{1}{4}}{100}$ is 115 $\frac{1}{4}$ times \$1, which is \$115 $\frac{1}{4}$.

21. What is the amount of \$50 for 2 years and 4 months, at 6%?

22. What is the amount of \$200 for 3 years and 4 months, at 9%?

23. What is the amount of \$40 for 4 years, at 5%?

24. What is the amount of \$75 for 3 years, at 4%?

25. What is the amount of \$70 for 2 years and 6 months, at 8%?

26. What is the amount of \$90 for 5 years, at 10%?

27. What is the amount of \$120 for 6 years and 3 months, at 8%?

28. What is the amount of \$64 for 8 years and 9 months, at $6\frac{1}{4}\%$?

29. What is the amount of \$192 for 3 years, 2 months and 15 days, at $12\frac{1}{2}\%$?

30. What is the amount of \$300 for 2 years, 5 months and 18 days, at 10%?

31. What is the amount of \$400 for 1 year, 7 months and 24 days, at 5%?

32. What is the amount of \$36 for 5 years, 10 months and 10 days, at 7%?

33. What principal will produce \$40 interest in 2 years and 6 months, at 8%?

ANALYSIS.—The interest at 8% for $2\frac{1}{2}$ years is 20%, or $\frac{1}{5}$ of the principal. If \$40 is $\frac{1}{5}$, 5 times \$40 or \$200 must be the principal.

34. What principal will produce \$60 interest in 3 years and 9 months, at 4%?

35. What principal will produce \$70 interest in 4 years, 8 months, at 6%?

36. What principal will produce \$105 interest in 5 years and 3 months, at 5%?

37. A house rents for \$250, which is 10% of its cost; what did the house cost?

38. B had a $\frac{1}{4}$ interest in a firm which made 20% a year on its capital, and his profits in 3 years, 3 months, were \$6500; what was the whole capital of the firm?

39. C has $\frac{2}{3}$ of his money invested at 8% interest, bringing him annually \$1600, and the remaining $\frac{1}{3}$ at 10% interest; what is his annual income from these investments, and what is the sum invested?

40. What principal will produce \$150 interest in 16 years, 8 months, at 6%?

41. What principal will produce \$1000 interest in 10 years, at 10%?

LESSON V.

1. What principal will amount to \$1200 in 2 years and 6 months, at 8 %?

ANALYSIS.—In 2 years, 6 months, at 8 %, the interest is 20 % or $\frac{1}{5}$ of the principal; therefore the amount is $\frac{6}{5}$ of the principal; if \$1200 is $\frac{6}{5}$ of the principal, $\frac{1}{6}$ of \$1200, which is \$200, must be $\frac{1}{5}$ of the principal, and 5 times \$200, or \$1000, must be the principal, which in 2 years, 6 months, at 8 %, amounts to \$1200.

2. What principal will amount to \$1000 in 5 years, at 5 per cent.?

3. What principal will amount to \$290 in 2 years and 8 months, at 6 %?

4. What principal will amount to \$600 in 3 years and 4 months, at 6 per cent.?

5. What principal will amount to \$270 in 5 years, at 7 per cent.?

6. What principal will amount to \$520 in 3 years and 4 months, at 9 %?

7. What principal will amount to \$1350 in 4 years and 2 months, at 12 per cent.?

8. After being associated 4 years, A and B dissolved partnership, when they found that their profits had averaged 10 % a year on their capital; A's share of capital was $\frac{2}{5}$, and his share of capital and profits amounted to \$28000. What were the capital and profits of both A and B?

9. A note amounted to \$150 in 8 years and 4 months, at 6 %; what was the face of the note?

10. C and D formed a partnership, D putting in only $\frac{1}{2}$ as much capital as C; at the end of 3 years they dissolved, having averaged an annual profit of $12\frac{1}{2}$ %. On dissolving, the united capital and profits amounted to \$33000; what were the capital and profits of both C and D?

11. If a 7 % bond and its coupons amount in 5 years and 6 months to \$1385, what is the face of the bond?

12. A certain principal at a certain per cent. amounted to \$93 in 4 years, and to \$102 in 6 years; what were the principal and rate per cent.?

13. What principal and rate will amount to \$270 in 5 years, and to \$312 in 8 years?

14. What principal and rate will amount to \$455 in 3 years, and to \$595 in 7 years?

NOTE.—The *present worth* of a debt payable at a future time is its worth at the present time, and is that principal which, at interest for the stated time and at the stated rate, will amount to the debt. Hence the present worth of a debt is found by the same analysis that is used in obtaining the principal from the amount, as in Example 1.

15. What is the present worth of a debt of \$26, payable 5 years hence, at 6%?

16. What is the present worth of \$81, due 5 years hence, at 7%?

17. What is the present worth of \$50, due 5 years hence, at 5%?

18. What is the present value of \$520, due 3 years and 9 months hence, at 8%?

19. What is the present value of \$390, due 3 years and 4 months hence, at 9%?

20. What is the discount on a note of \$103, due 6 months hence, at 6%?

NOTE.—The *discount* is the difference between the sum promised and its present worth. Discount is a deduction from a debt, allowed for the payment of the debt before it is due.

21. What is the discount of \$53 due in 1 yr., at 6%?

22. What is the discount of \$51 due in 3 mo., at 8%?

23. What is the discount of \$175 due in 1 year and 8 months, at 10%?

24. When money is worth 9% a year, if I buy a note 8 months before it is due, how much less than \$212, its face, should I give for it?

25. When money is worth 1% a month, what should be given for a note, whose face is \$545, having 9 months to run? What is the discount?

LESSON VI.

BANK DISCOUNT is the interest of the face of a note, deducted from the face at the time of the loan. The remainder is called the *proceeds* of the note.

1. What are the proceeds of a note for \$100 running 93 days at 6%?

ANALYSIS.—Calling 93 days 93 mills, $\frac{1}{8}$ of 93 mills is $15\frac{1}{8}$ mills, and 100 times $15\frac{1}{8}$ mills are 1550 mills, or \$1.55. \$100 less \$1.55 is \$98.45. Therefore the proceeds of \$100 for 93 days at 6% are \$98.45.

2. What are the proceeds of a note for \$200 running 93 days at 6%?

3. What are the proceeds of a note for \$300 running 63 days at 6%?

4. What are the proceeds of a note for \$50 running 123 days at 6%?

5. What are the proceeds of \$80 running 33 days at 9%? (NOTE.—9% is $\frac{3}{2}$ of 6%.)

6. What are the proceeds of a note for \$120 running 183 days at 7%? 8%? 5%? 10%?

7. What are the proceeds of a note for \$400 running 153 days at 10%? 11%? 12%? 15%?

8. At 6%, in what time will \$50 gain \$12?

FIRST ANALYSIS.—The interest \$12 is $\frac{12}{50}$ of the principal. If in 1 year the interest is $\frac{6}{100}$ or $\frac{3}{50}$ of the principal, it will require as many years to gain $\frac{12}{50}$ of the principal as $\frac{3}{50}$ is contained times in $\frac{12}{50}$. Since $\frac{3}{50}$ is contained 4 times in $\frac{12}{50}$, \$50 at 6% will gain \$12 in 4 years.

SECOND ANALYSIS.—Since \$50 at 6% gains \$3 in 1 year, it will require as many years to gain \$12 as \$3 is contained times in \$12. Since \$3 is contained 4 times in \$12, \$50 at 6% will gain \$12 in 4 years.

9. At 6%, in what time will \$125 gain \$45?

10. At 5%, in what time will \$40 gain \$10?

11. At 5%, in what time will \$75 gain \$30?

12. At 4%, in what time will \$500 gain \$140?

13. At 7%, in what time will \$200 gain \$91?
14. At 8%, in what time will \$150 gain \$40?
15. At 9%, in what time will \$300 gain \$45?
16. At 10%, in what time will \$160 gain \$100?
17. At 12%, in what time will \$200 gain \$100?
18. At 5%, in what time will any principal double itself?

ANALYSIS.—At 5%, any principal will gain 100% of itself, that is, double itself, in as many years as 5% is contained times in 100%. Since 5% is contained 20 times in 100%, any principal at 5% will double itself in 20 years.

19. At 10%, in what time will any principal double itself? At 4%? 8%? 20%? 25%?

20. At 6%, in what time will any principal double itself? At 15%? $12\frac{1}{2}\%$? $6\frac{1}{4}\%$?

21. At $16\frac{2}{3}\%$, in what time will any principal double itself? At 12%? At 3%?

22. At 7%, in what time will any principal double itself? At 2%? 1%? 11%?

23. At what per cent. will \$80 yield \$10 in $2\frac{1}{2}$ years?

FIRST ANALYSIS.—If the interest for $2\frac{1}{2}$ yrs. is \$10, that for 1 year is as many dollars as $2\frac{1}{2}$ is contained times in 10, that is, \$4. Since \$4 is $\frac{4}{80}$ or $\frac{1}{20}$, which is $\frac{5}{100}$, or 5%, of the principal, \$80 dollars will yield \$10 in $2\frac{1}{2}$ years at 5%.

SECOND ANALYSIS.—At 1%, the interest of \$80 for $2\frac{1}{2}$ yrs. is \$2. If \$2 is produced by 1%, \$10 will be produced by as many per cent. as 2 is contained times in 10, that is, by 5%. Therefore \$80 will yield \$10 in $2\frac{1}{2}$ years at 5%.

24. At what per cent. will \$50 yield \$10 in 3 years and 4 months?

25. At what per cent. will \$100 yield \$35 in 5 years? In 5 years and 10 months?

26. At what per cent. will \$125 yield \$45 in 4 years and 6 months?

27. At what per cent. will \$150 yield \$50 in 3 years and 4 months?

28. At what per cent. will a principal double itself in 5 yrs.? 10? 20? $12\frac{1}{2}$? $16\frac{2}{3}$? 25? $33\frac{1}{3}$? 50?

CHAPTER XII.

MISCELLANEOUS EXERCISES.

LESSON I.

1. JAMES, having a certain number of cents, earned as many more, and then had 30 cents; how many cents had he at first?

ANALYSIS.—When he earned as many more as he had, he had twice as many as he had at first. If twice that number equals 30, that number must be $\frac{1}{2}$ of 30, or 15. Therefore he had 15 cents at first.

2. John has as much money as Henry, and they have in all 50 cents; how much has each?

3. What number added to itself equals 190?

4. The united wages of a man and his son are \$2.25, and the man's wages are twice as great as those of his son. What are the wages of each?

5. What number added to twice itself equals 75?

6. The killed and wounded in a certain battle were 500, and the number of wounded was 3 times that of the killed; how many were there of each?

7. What number added to 4 times itself equals 450?

8. The united ages of a man and a boy are 72 years, and the man is 5 times as old as the boy; how old is each?

9. What number added to 6 times itself equals 287?

10. A man's crops of wheat and rye amounted to 400 bushels, and the wheat crop was 7 times the rye crop; how many bushels were in each crop?

11. What number added to 8 times itself equals 72?

12. What number added to 11 times itself equals 42?

13. If a certain person lives $\frac{1}{2}$ as long as he has lived, he will be 60 years old; how old is he?

ANALYSIS.—If he lives $\frac{1}{2}$ as long in future as he has lived, he will then be $1\frac{1}{2}$, or $\frac{3}{2}$, times as old as he is now. If 60 years are $\frac{3}{2}$ of his present age, $\frac{1}{3}$ of 60, or 20, is $\frac{1}{2}$ of his present age, and 2 times 20, or 40, must be his present age. Therefore he is 40 years old.

14. Two schools have 80 pupils, and one has $\frac{1}{3}$ as many as the other; how many has each?

15. What number added to $\frac{1}{4}$ of itself equals 75?

16. What number added to $\frac{3}{4}$ of itself equals 84?

17. The united ages of A and B are 81 years, and A's age is $\frac{4}{5}$ of B's; how old is each?

18. A drover has $\frac{2}{3}$ as many horses as oxen, in all 100 animals; how many of each kind has he?

19. An army of 14000 men has $\frac{1}{8}$ as many cavalry as infantry; how many of each kind has it?

20. After Richard had won $\frac{7}{8}$ as many marbles as he had at first, he had 30; how many had he at first?

21. What number is that, $\frac{2}{3}$ of which added to $\frac{1}{4}$ of it equals 121?

NOTE.— $\frac{2}{3} + \frac{1}{4}$ are $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$.

22. What number is that, $\frac{3}{5}$ of which added to $\frac{1}{10}$ of it equals 42?

23. What number is that, which added to its third and $\frac{1}{2}$ of its sixth, equals 51?

24. A, B, and C are together worth \$4800; A is worth $\frac{2}{3}$ as much as B, and C is worth $\frac{1}{2}$ as much as A; how much is each worth?

25. Three farmers, D, E, and F, own in all 680 acres; E owns $\frac{4}{5}$ as much as D, and D owns $\frac{5}{8}$ as much as F; how many acres does each own?

26. Three farmers raised each the same number of bushels of wheat; $\frac{2}{3}$ of the crop of the first, with $\frac{3}{4}$ of that of the second and $\frac{4}{5}$ of that of the third, were 266 bushels; what was the crop raised by each?

27. The sum of four numbers is 250; the first is $\frac{1}{4}$ of the second, the second is $\frac{1}{2}$ of the third, and the third is $\frac{3}{4}$ of the fourth; what are the numbers?

LESSON II.

1. The sum of two numbers is 22, and $\frac{2}{3}$ of the first equals $\frac{4}{5}$ of the second; what are the numbers?

ANALYSIS.—If $\frac{2}{3}$ of the first equals $\frac{4}{5}$ of the second, $\frac{1}{3}$ of the first equals $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{2}{5}$, of the second, and $\frac{3}{3}$ or the whole of the first equals 3 times $\frac{2}{5}$, or $\frac{6}{5}$, of the second. Therefore $\frac{5}{5} + \frac{6}{5}$, or $1\frac{1}{5}$, of the second equals 22. If $\frac{1}{5}$ of the second is 22, $\frac{1}{11}$ of 22, or 2, is $\frac{1}{5}$ of the second, and 5 times 2, or 10, must be the second. Since the first is $\frac{6}{5}$ of the second, $\frac{6}{5}$ of 10, which is 12, must be the first.

2. Walter and William together have 36 cents, and $\frac{1}{4}$ of Walter's money is $\frac{1}{5}$ of William's; how many cents has each?

3. The united ages of Alice and Ellen are 28 years, and $\frac{2}{3}$ of Alice's age is $\frac{1}{3}$ of Ellen's; how old is each?

4. George and Charles bought knives; $\frac{3}{5}$ of the cost of George's knife was $\frac{2}{5}$ of the cost of Charles's; both knives cost \$1.25; what was the cost of each?

5. In an orchard of 48 apple-trees and pear-trees, $\frac{5}{6}$ of the pear-trees equal $\frac{1}{2}$ of the apple-trees; how many trees of each kind are in the orchard?

6. A watch and chain cost \$100, and $\frac{1}{3}$ of the cost of the watch equals $\frac{6}{7}$ of the cost of the chain; what was the cost of each?

7. In two pastures are 300 sheep, and $\frac{5}{8}$ of the number in one pasture equals $\frac{5}{12}$ of the number in the other; how many sheep are in each pasture?

8. A horse and harness cost \$150, and $\frac{4}{5}$ of the cost of the horse was $3\frac{1}{5}$ times the cost of the harness; what was the cost of each?

9. What two numbers, whose sum is 46, are to each other as $\frac{2}{5}$ to $\frac{3}{4}$?

ANALYSIS.— $\frac{2}{5}$ equals $\frac{8}{20}$, and $\frac{3}{4}$ equals $\frac{15}{20}$; therefore $\frac{2}{5}$ is to $\frac{3}{4}$ as 8 to 15. If 46 is composed of 8 parts + 15 parts, that is, of 23 parts, $\frac{1}{23}$ of 46, which is 2, is one of those parts, and 8 times 2, or 16, is one of the required numbers, and 15 times 2, or 30, is the other.

10. What two numbers, whose sum is 121, are to each other as $\frac{5}{8}$ and $\frac{3}{4}$?

11. Divide 38 into parts which shall be to each other as $3\frac{1}{2}$ to $1\frac{1}{4}$.

12. Divide 69 into three parts which shall be to each other as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

13. Divide 85 into three parts which shall be to each other as $1\frac{1}{4}$, $1\frac{1}{6}$, and $1\frac{1}{8}$.

14. A, B, and C shared 90 horses as they had furnished the money to buy them. As often as A had furnished \$2, B had furnished $\$2\frac{1}{2}$, and C \$3. How many horses did each receive?

15. D, E, and F divide \$840, of which D gets \$200; how much do E and F get, if E had furnished \$600 and F \$1000; and how much had D furnished, if all had been partners an equal time?

16. A certain man is 45 years old, which is 11 years more than twice his son's age; what is his son's age?

ANALYSIS.—If 45 is 11 years more than twice his son's age, $45 - 11$, or 34, must be twice his son's age, and $\frac{1}{2}$ of 34, or 17, must be his son's age.

17. Three times a number and 6 more equal 75; what is that number?

18. What number increased by its half, third, fourth, sixth, and 9 more, equals 90?

19. From 4 times a number take 6, and the remainder is 50; what is that number?

20. If from a number its third and its fourth be taken and the remainder increased by 15, the result is 40; what is that number?

21. If twice a number be diminished 25%, and the remainder be diminished by 25, the result is the number; what is the number?

22. If $\frac{1}{3}$ of a number be increased 50%, and from this result 50 be taken, the result is $\frac{1}{6}$ of the number; what is the number?

23. If 40% of a number be multiplied by $2\frac{1}{8}$, and from the product 70 be taken, the result is $\frac{1}{2}$ of the number; what is the number?

LESSON III.

1. Lewis and Stephen together had 61 cents, and Lewis had 11 more than Stephen; how many had each?

ANALYSIS.—Since Lewis had 11 cents more than Stephen had, both had 11 cents more than twice what Stephen had. Since both had 61 cents, $61 - 11$, or 50 cents, is twice what Stephen had, and $\frac{1}{2}$ of 50 cents, or 25 cents, is what Stephen had, and Lewis had $25 + 11 = 36$ cents.

2. Mary and Jane together have 30 pins, and Jane has 8 more than Mary has; how many has each?

3. Two horses cost \$180, and one cost \$20 less than the other; what did each cost?

4. Three melons cost together 50 cents; the second cost 5 cents more than the first, and the third 10 cents more than the second; what did each cost?

5. A calf, a cow, and a colt were sold for \$55; the colt brought \$20 less than the cow, and the calf \$10 less than the colt; what did each bring?

6. In two lots are 52 acres, and in one are 10 acres more than $2\frac{1}{2}$ times the other; how many in each?

7. Two farmers have 165 sheep, and one has 5 less than $3\frac{1}{4}$ times what the other has; how many has each?

8. If from $5\frac{1}{2}$ times a number you take 120, the remainder is 4 times the number; what is the number?

9. If to $1\frac{3}{4}$ times a number you add 50, the sum is $4\frac{1}{4}$ times the number; what is the number?

10. Hall had $\frac{1}{4}$ as many cents as Martin; after Martin had spent 10 cents, they together had 30 cents; how many had each at first?

11. Hiram had found 3 times as many eggs as Horace; after Hiram had found 7 more and Horace 8, they had together 75; how many had they at first?

12. Amos and Jarvis together earned \$5, but Jarvis earned 50 % more than Amos; what did each earn?

13. Two farmers raised together 600 bushels of oats, but one raised 50 % less than the other; how many bushels of oats did each raise?

14. Herman, Henry, and Howell had together 54 cents; Henry had twice as many as Herman, and Howell had three times as many as Henry; how many cents had each?

ANALYSIS.—Since Henry has twice Herman's money, they two have three times Herman's money, and since Howell has 3 times Henry's money, he has 6 times Herman's, and all have $6 + 3 = 9$ times Herman's money. Since 54 cents is 9 times Herman's money, $\frac{1}{9}$ of 54 cents, or 6 cents, is his money, and 2×6 cts, or 12 cts., is Henry's, and 3×12 , or 36 cts., is Howell's.

15. Divide 100 into 3 such parts that the second shall be 4 times the first, and the third 5 times the second.

16. Divide 55 into 3 such parts that the first shall be $\frac{1}{2}$ of the second, and the second $\frac{1}{4}$ of the third.

17. Divide 324 into 3 such parts that $\frac{1}{2}$ of the first is twice the second, and $\frac{1}{4}$ of the second 4 times the third.

18. Divide 39 into 3 such parts that $1\frac{1}{2}$ times the first equals the second, and $1\frac{1}{2}$ times the second, and 1 more, equals the third.

19. A, B, and C paid \$250 for a lease; $\frac{1}{3}$ of what A paid, and \$5 more, is what B paid, and $\frac{1}{6}$ of what A paid, and \$20 more, is what C paid; what did each pay?

20. A father divided his estate among his 3 sons, so that the first had \$1000 less than $\frac{1}{2}$ of the whole, the second \$800 less than $\frac{1}{3}$ of the whole, and the third \$600 less than $\frac{1}{4}$ of the whole; what was the estate, and how much did each son receive?

21. If a certain number is divided by 8, the sum of the number, the quotient, and divisor, is 116; what is the number?

22. An estate is divided among 4 children, so that the first has \$200 more than $\frac{1}{4}$ of the whole, the second \$300 more than $\frac{1}{6}$ of the whole, the third \$500 more than $\frac{1}{8}$ of the whole, and the fourth \$800 more than $\frac{1}{12}$ of the whole; what was the estate, and how much has each?

23. What two numbers, whose difference is 20, are to each other as 5 to 4?

24. What number is that, $\frac{3}{5}$ of which exceeds $\frac{2}{3}$ of it by 36?

LESSON IV.

1. If a barrel of flour lasts one family 8 weeks and another 6 weeks, how long would it last both?

ANALYSIS.—One family consumes $\frac{1}{8}$ of the flour in a week, the other $\frac{1}{6}$ of it in a week; both consume $\frac{1}{8} + \frac{1}{6} = \frac{3}{24} + \frac{4}{24} = \frac{7}{24}$ of it in a week. If $\frac{7}{24}$ of it last both families 1 week, $\frac{24}{7}$, or the whole, would last as many weeks as $\frac{7}{24}$ is contained times in $\frac{24}{7}$. Since 7 is contained in 24 $3\frac{3}{7}$ times, the barrel would last both families $3\frac{3}{7}$ weeks.

2. If a man can hoe a field in 4 days, and a boy in 6 days, how long would it take them working together?

3. If one team can plow a field in 3 days, and another in 4 days, how long would it take both to plow it?

4. If two women can each do the washing in a day, how long would it take both together?

5. If one compositor can set a certain amount of matter in 10 hours, and another in 12 hours, how long would it take both together to set it?

6. One pipe can fill a cistern in $2\frac{1}{2}$ hours, and another pipe can fill it in 3 hours; how long would both pipes be in filling it?

7. Two pipes can fill a cistern in 3 hours, and one of them can fill it in 5 hours; in what time can the other pipe fill the cistern?

ANALYSIS.—Both fill $\frac{1}{3}$ of it in an hour, and one fills $\frac{1}{5}$ of it in an hour; therefore the other fills $\frac{1}{3} - \frac{1}{5} = \frac{5}{15} - \frac{3}{15} = \frac{2}{15}$ of it in an hour. If it fills $\frac{2}{15}$ in 1 hour, it will take as many hours to fill $\frac{15}{2}$, or the whole, as 2 is contained times in 15. Since $15 \div 2 = 7\frac{1}{2}$, it will take the second pipe $7\frac{1}{2}$ hrs. to fill the cistern.

8. If 2 men can reap a field in 6 hours, and one of them can do it in 10 hours, how long would it take the other?

9. If 2 men earn \$100 a month, and one of them earns as much in 3 months, how long would it take the other?

10. If a certain quantity of provisions last 2 men 8 days, and would last one of them 12 days, how long would they last the other?

11. If two choppers can cut down the trees of a certain

wood in 4 weeks, and it would take one of them 7 weeks, how long would it take the other?

12. If 2 men can mow the grass of a certain field in 10 hours, and it would take one of them 16 hours, how long would it take the other?

13. A and B can dig a cellar in 6 days, A and C in 8 days, and B and C in 12 days; how long would it take each to dig it alone?

ANALYSIS.—A and B dig $\frac{1}{6}$ of it in a day, A and C $\frac{1}{8}$ of it in a day; therefore 2 A's with B and C can dig $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$ of it in a day; but B and C dig $\frac{1}{12}$ of it in a day; therefore 2 A's dig $\frac{7}{24} - \frac{1}{12} = \frac{5}{24}$ in a day, and A alone digs $\frac{1}{2}$ of $\frac{5}{24} = \frac{5}{48}$, in a day. If A and B dig $\frac{1}{6}$, B digs $\frac{1}{6} - \frac{5}{48} = \frac{3}{48}$, or $\frac{1}{16}$, in a day. If B and C dig $\frac{1}{12}$, C digs $\frac{1}{12} - \frac{1}{16} = \frac{1}{48}$. If A digs $\frac{5}{48}$ in a day, the whole would take him $48 \div 5 = 9\frac{3}{5}$ days. If B digs $\frac{1}{16}$ in a day, the whole would take him 16 days. If C digs $\frac{1}{48}$ in a day, the whole would take him 48 days.

14. A and B can mow a field in 2 hours, A and C in 3 hours, and B and C in 4 hours; how long would it take each to mow it alone?

15. A and B can dip up a hogshhead of water in 8 minutes, A and C in 9 minutes, and B and C in 10 minutes; how long would it take each alone?

16. A, B, and C can dig a ditch in 4 days, A and B in 6 days, and B and C in 8 days; how long would it take each separately to dig the ditch?

17. A, B, and C, carpenters, can finish a house in 2 months, A with B can do it in $2\frac{1}{2}$ months; how long would it take C to do it alone?

18. If Jane can do a certain job of sewing in $\frac{3}{5}$ of a day, and Sarah in $\frac{2}{3}$ of a day, in what part of a day would they do it working together?

19. John can hoe the potato-patch in $1\frac{1}{2}$ hours, and James in 1 hour. In what time can John finish the job after James has hoed $\frac{1}{3}$ of an hour? In what time can James finish it after John has hoed $\frac{1}{2}$ of an hour?

20. If A does $\frac{7}{8}$ as much as B, and C does $\frac{2}{5}$ as much as A and B, and all do a job in 7 days, in how many days could each do it working alone?

LESSON V.

1. A merchant sold cloth at \$2 a yard, and by so doing lost \$6; if he had sold it at \$4 a yard, he would have gained \$18; how many yards did he sell?

ANALYSIS.—Since \$2 more on a yd. makes $\$6 + \$18 = \$24$ more on the whole, he sold as many yds. as \$2 is contained times in \$24; that is, he sold 12 yds.

2. A boy bought some oranges for 3 cents apiece; if he had given 5 cents apiece, they would have cost 20 cents more; how many did he buy?

3. Some men raised \$48 by a subscription; had they each given \$5 more, they would have raised \$88; how many men were there?

4. A train moved from one city to another in 10 hours; had it gone 20 miles per hour faster, it would have performed the journey in half the time; what was the distance between the cities?

5. If, by buying a certain number of apples at 2 cents apiece, I should have 7 cents left, and buying them at 3 cents apiece would take 2 cents more than I have, how many cents have I?

6. A and B walk to meet each other, A at 3 miles per hour, and B at 4 miles per hour; if each should walk $\frac{1}{2}$ mile per hour faster, they would meet 2 hours sooner; how far apart are they?

7. What number is that whose product when multiplied by 9 is 28 more than its product by 5?

8. Eight times a certain number is 50 less than 13 times the number; what is the number?

9. A boy said that 2 times his age was 25 years less than 4 times his age; according to that, how old was he?

10. What number is that $\frac{2}{3}$ of which is 7 more than one-half of it?

11. If $\frac{3}{5}$ of a number is 9 less than $\frac{3}{4}$ of it, what is it?

12. If A is 4 times as old as B, B $\frac{1}{2}$ as old as C, and C is 30 years younger than A, how old is each?

13. A man traveled 74 miles in 3 days; the first day he traveled $\frac{4}{3}$ as far as the second, and the third day $\frac{3}{4}$ as far as the second; how far did he travel each day?

ANALYSIS.—He traveled in the first and third days $\frac{4}{3} + \frac{3}{4} = \frac{16}{12} + \frac{9}{12} = \frac{25}{12}$ as far as the second day; therefore in the three days he traveled $\frac{25}{12} + \frac{12}{12} = \frac{37}{12}$ as far as the second day. If 74 miles is $\frac{37}{12}$ of the second day's travel, $\frac{1}{37}$ of 74 miles, or 2 miles, is $\frac{1}{12}$, and 12 times 2 miles, or 24 miles, is his second day's travel; $\frac{4}{3}$ of 24 = 32 miles, his first day's travel, and $\frac{3}{4}$ of 24 = 18 miles, his third day's travel.

14. A boat sailed 27 miles in 3 hours; the second hour it went $\frac{5}{6}$ as far as the first, and the third hour $\frac{1}{2}$ as far as the second; how far did it go each hour?

15. A flag-pole was made of 3 pieces. The top was 25 feet long, the middle as long as the top and $\frac{2}{3}$ of the bottom, and the bottom as long as both the other pieces; what was the length of the middle piece?

16. A pole, 100 feet high, stands 4 feet in the ground, and in the water 5 feet more than $\frac{1}{12}$ of the length of that part in the air; what length is in the water?

17. The skin of an ox weighed $\frac{5}{9}$ of the weight of each fore-quarter, and each hind-quarter weighed $\frac{8}{5}$ of the weight of the skin. The skin and quarters weighed in all 780 lbs.; what did each part weigh?

18. Of three numbers, the first is 15, the second is equal to the first and $\frac{2}{3}$ of the third, and the third is equal to the first two; what is the sum of the numbers?

19. Four towns are situated in the order of the letters A, B, C, and D. The distance from A to D is 51 miles; the distance from A to B is to that from C to D as 2 to 3; and $\frac{1}{4}$ of that from A to B, increased by $\frac{1}{2}$ of that from C to D, is 3 times that from B to C; what are the distances?

20. The sum of the ages of A, B, and C is 72 years; $\frac{1}{2}$ of A's age, $\frac{1}{3}$ of B's, and $\frac{1}{4}$ of C's equal each other; what are their ages?

21. What number is that which, when added to 18 and 26, will make the former sum to the latter as 3 to 4?

22. What number is that, $\frac{7}{12}$ of 120% of which is 35?

23. What number is that, $\frac{2}{5}$ of 62 $\frac{1}{2}$ % of which is 23?

LESSON VI.

1. A man distributed 20 cents among an equal number of boys and girls, giving to each girl 3 cents and to each boy 2 cents; to how many persons did he give?

ANALYSIS.—A boy and a girl received 5 cents; therefore there were as many such couples as 5 is contained times in 20; therefore there were 4 couples, that is, 4 boys and 4 girls.

2. The wages of an equal number of men and boys in a factory amounted to \$360 a week, each man earning \$9 and each boy \$3; how many of each were there?

3. A man, with his son and his horse, worked for a neighbor, at the rate of \$1.50 per day for the man, \$2 for the horse, and 50 cts. for the boy. At the end of the service \$50 were due; how many days did they work?

4. If $\frac{2}{3}$ of a number of boys have 10 cents apiece, and the rest 5 cents apiece, and all have \$1.50, what is the number of boys?

5. If $\frac{3}{4}$ of a number of lemons cost 2 cents apiece, $\frac{1}{2}$ of the rest 3 cents apiece, and the remainder 4 cents apiece, how many are there, if all cost 57 cents?

6. A drover gave \$500 for a number of cows, sheep, and hogs. The number of cows was $\frac{1}{3}$ of that of the sheep, and the number of sheep was $\frac{1}{2}$ of that of the hogs; how many of each were there, if the sheep were \$2 a head, the hogs \$4 a head, and the cows \$20 a head?

7. Going to market for a certain number of pounds of meat, I found that if I bought mutton at 6 cents, I should have 6 cents left, but if I bought beef at 8 cents, I should not have enough to pay for it; so I bought $\frac{2}{3}$ of the quantity in mutton, and the rest in beef, which just took my money; how many pounds did I buy?

8. If 3 times a number is 4 more than 4 times $\frac{2}{3}$ of it, what is that number?

9. If 8 times a number is 5 more than 3 times $\frac{4}{5}$ of it, plus 4 times $\frac{9}{10}$ of it, plus 2 times $\frac{3}{4}$ of it, what is that number?

10. A man's boarding was \$1 per day, and his wages were \$3 per day. At the end of 60 days, he had made only \$90; how many days was he idle?

ANALYSIS.—Since every day he worked, he made \$3 wages less \$1 boarding, if he had worked 60 days he would have made \$120: since he made only \$90, he lost \$30 by idleness. Every idle day he lost \$3; therefore, to lose \$30, he must have been idle as many days as \$3 is contained times in \$30, that is, 10 days.

11. Henry's wages are \$2 a day, and he pays 50 cents a day for board; if in 30 days he gets \$35 above his board, how many days is he idle?

12. A man contracted to dig a well for \$4 a day, on condition that he should forfeit \$5 for every day he was idle; in 12 days he finished the well, and received \$30; how many days was he idle?

13. A boy carried 48 wine-glasses to a house, for a cent apiece, on condition that he should forfeit 5 cents to pay for every one he broke. He received 30 cents; how many glasses did he break?

14. A journeyman shoemaker agreed to make boots for \$1.25 per pair, on condition that he should pay \$1.75 per pair for the leather of every pair spoiled. After working on 20 pairs, his due was \$19; how many pairs of boots did he spoil?

15. A merchant bought 100 barrels of flour on these conditions: he was to pay \$5 per barrel for all that was good, and \$1 per barrel for all that was injured; at settlement he paid \$440; how many barrels were injured?

16. A miller sold 50 barrels of flour of two qualities; for the better quality he got \$5.50 per barrel, and for the other \$3.75 per barrel; for all he got \$240; how many of each quality were sold?

17. A manufacturer of shoes sold 400 pairs of boots and shoes for \$900; the boots averaged \$3.50 a pair, and the shoes \$1.50 a pair; how many pairs of each were sold?

18. A copyist transcribed 150 pages for 10 cents per page, forfeiting 10 cents for each error; he received \$10; how many errors did he make?

LESSON VII.

1. A father is 30 years old, and his son 5 years old; in how many years will the son's age be $\frac{1}{2}$ of the father's?

ANALYSIS.—When the son was born, the father was $30 - 5 = 25$ years old; therefore both must live 25 yrs. from the son's birth, to make the son's age $\frac{1}{2}$ of the father's; since the son is now 5 yrs. old, he has only 20 yrs. of the 25 to live to make his age $\frac{1}{2}$ of his father's age.

2. A is 28 years old, and B is 8; in how many years will B's age be $\frac{1}{2}$ of A's age?

3. If A's age is 28 years, and B's is 8, in how many years will B's age be $\frac{1}{3}$ of A's age?

NOTE.—Reckon from B's birth, $\frac{1}{2}$ of A's age at B's birth.

4. Joseph is 17 years old, and his youngest brother is 2 years old; in how many years will Joseph be 4 times as old as his brother?

5. Albert is 15 years old, and his sister is 7 years old; how many years ago was Albert 5 times as old as his sister was?

6. A man is 38 years old, and his wife is 33 years old; how old was his wife, when his age was 6 times hers?

7. A teacher is 30 years old, and his pupil 12; how long ago was the teacher 7 times as old as the pupil?

8. Mr. Rogers is 48 years old, and his son is 13; how long ago was he 8 times as old as his son?

9. Mr. Williams is 3 times as old as his son, who is 16 years of age; what was Mr. Williams's age when he was 9 times as old as his son?

10. James is 15 years old, which is $\frac{1}{4}$ of his uncle's age; how long since James was $\frac{1}{10}$ as old as his uncle?

11. Lucy's age is to Sarah's as 3 to 2, and the sum of their ages is 25 years; when will Sarah's age be to Lucy's as 5 to 6, and what will then be the sum of their ages?

12. Two trains going the same way, each 10 miles an hour, pass a station, one 54 minutes behind the other; how many minutes before the forward train is 4 times as far from the station as the other?

13. A person being asked the hour, said, "The time since 6 o'clock this morning is $\frac{5}{7}$ of the time to noon;" what o'clock was it?

ANALYSIS.—The time since 6 o'clock being $\frac{5}{7}$ of that to noon, both times are $\frac{5}{7} + \frac{2}{7} = \frac{1}{1}$ of that to noon; therefore $\frac{1}{7}$ of the time to noon is $\frac{1}{12}$ of the 6 hours from 6 o'clock till noon, which is $\frac{1}{2}$ of an hour, and $\frac{5}{7}$ are $\frac{5}{2}$ of an hour, that is, $2\frac{1}{2}$ hours; therefore the time of day was 6 o'clock + $2\frac{1}{2}$ hrs. = $8\frac{1}{2}$ o'clock, A.M.

14. What is the hour, if the time since 7 o'clock A.M. is $\frac{2}{3}$ of the time to noon?

15. What is the hour, if the time since noon is $\frac{3}{5}$ of the time to 6 o'clock P.M.?

16. What o'clock is it, if the time since 3 o'clock P.M. is $\frac{4}{5}$ of the time to midnight?

17. What o'clock is it, if $\frac{1}{2}$ of the time since midnight is $\frac{1}{3}$ of the time to 10 o'clock A.M.?

18. What is the hour, if $\frac{2}{3}$ of the time since 10 o'clock A.M. is the time to 3 o'clock P.M.?

19. From the ground to the top of a church-spire is 108 feet, and $1\frac{3}{4}$ times the height of the church is $1\frac{2}{3}$ times the length of the spire; what are the height of the church and the length of the spire?

20. How many gallons are there in a barrel whose capacity is 40 gallons, when $2\frac{1}{4}$ times the filled space is $2\frac{3}{4}$ times the space unfilled?

21. A tailor being asked how much cloth remained of a certain piece of 35 yards, said, " $3\frac{1}{2}$ times what I have cut off is $4\frac{2}{3}$ times the remainder." How much remained?

22. A traveler, inquiring how far it was to an inn, was told that 5 times what it lacked of 10 miles was $3\frac{1}{3}$ times what it exceeded 5 miles; how far was it to an inn?

23. What o'clock is it, when $1\frac{2}{3}$ the time since 9 o'clock A.M. is the time to 9 o'clock P.M.?

24. What o'clock is it, when $\frac{1}{3}$ of the time since 6 o'clock A.M. is $\frac{1}{9}$ of the time since midnight?

25. What is the hour, when $\frac{1}{2}$ of the time since 6 o'clock P.M. is $\frac{1}{11}$ of the time since midnight?

26. How old is he who lacks $\frac{3}{4}$ of his present age of being 70 years old?

LESSON VIII.

1. At what time after 2 o'clock are the hour-hand and minute-hand of a clock together?

ANALYSIS.—The minute-hand goes 12 hour-spaces while the hour-hand goes 1; therefore at 1 o'clock the minute-hand has one space to gain to overtake the hour-hand. If in going 12 hour-spaces, the minute-hand gains 11 spaces, to gain 1 more space, it must go $\frac{1}{11}$ of 12 hour-spaces, which is $1\frac{1}{11}$ hour-spaces. Therefore the hour-hand and minute-hand are together at $5\frac{5}{11}$ minutes past 1 o'clock. At 2 o'clock the minute-hand has 2 hour-spaces to gain to overtake the hour-hand, and it must go $\frac{2}{11}$ of 12 hour-spaces to gain them; therefore the hands are again together at $10\frac{10}{11}$ minutes past 2 o'clock.

2. At what time after 3 o'clock are the hour-hand and minute-hand of a clock together?

3. At what time after 4 o'clock are the hour-hand and minute-hand of a clock together?

4. At what time after 5 o'clock are the hour-hand and minute-hand of a clock together?

5. At what time after 6 o'clock are the hour-hand and minute-hand of a clock together?

6. At what time between 1 o'clock and 2 o'clock are the hour-hand and minute-hand exactly opposite?

NOTE.—In this case the minute-hand must gain 6 spaces *beyond* the hour-hand; that is, it must gain 7 spaces, and must go $\frac{7}{11}$ of 12 spaces, $= 7\frac{7}{11}$ spaces; therefore the hands are opposite at $7\frac{7}{11}$ times 5 minutes, *i.e.* $38\frac{2}{11}$ minutes past 1 o'clock.

7. At what time between 2 o'clock and 3 o'clock are the hour-hand and minute-hand exactly opposite?

8. At what time between 3 o'clock and 4 o'clock are the hour-hand and minute-hand exactly opposite?

9. At what time between 4 o'clock and 5 o'clock are the hour-hand and minute-hand exactly opposite?

10. At what time between 7 o'clock and 8 o'clock are the hour-hand and minute-hand exactly opposite?

NOTE.—In this case 7 *less* 6 spaces, or 1 space, is to be gained.

11. Find the time of opposition of hands between 8 o'clock and 9 o'clock; 9 o'clock and 10 o'clock; 10 o'clock and 11 o'clock.

12. The united ages of A and B are 70 years; and $\frac{2}{3}$ of A's age, plus 12 years, is $\frac{4}{5}$ of B's age; how old is each?

ANALYSIS.—If $\frac{4}{5}$ of B's age is 12 years more than $\frac{2}{3}$ of A's age, $\frac{1}{5}$ of B's age is $\frac{1}{4}$ of $(12 \text{ yr.} + \frac{2}{3}) = 3 \text{ yr.} + \frac{1}{6}$ of A's age, and $\frac{5}{6}$ of B's age must be 15 years $+ \frac{5}{6}$ A's age. Therefore $\frac{6}{6}$, or the whole of A's age, $+ \frac{5}{6}$ A's age $+ 15$ years, are 70 years, and $\frac{11}{6}$ A's age $= 70 - 15 = 55$ years, and $\frac{1}{6}$ A's age is $\frac{1}{11}$ of 55 yrs., or 5 yrs. Therefore A's age must be $6 \times 5 = 30$ yrs., and B's $70 - 30 = 40$ yrs.

13. C and D have together \$38, and $\frac{2}{3}$ of C's money is \$4 more than $\frac{2}{5}$ of D's; how much has each?

14. What is the hour, when $\frac{2}{3}$ of the time past noon is $2\frac{2}{3}$ hrs. more than $\frac{2}{9}$ of the time to midnight?

15. A beam 60 feet long was cut into two pieces, so that 10 ft. more than $\frac{3}{4}$ of the less part was $\frac{5}{8}$ of the greater part; how long were the pieces?

16. The sum of two numbers is 90, and 12 more than $\frac{3}{5}$ of one number is $\frac{1}{2}$ of the other; what are the numbers?

17. The sum of two numbers is 150, and 15 less than $\frac{5}{8}$ of one number is $\frac{1}{2}$ of the other; what are the numbers?

18. The sum of two numbers is 120, and 20 less than $\frac{2}{3}$ of one number is $\frac{2}{3}$ of the other; what are the numbers?

19. The killed and wounded in a certain battle were 225, and 15 less than $\frac{9}{10}$ of the killed equaled $\frac{3}{5}$ of the number of wounded; what was the number of each?

20. There are 45 pupils in a certain school, and 10 more than $2\frac{1}{2}$ times the number of boys equals $2\frac{2}{5}$ times the number of girls; what is the number of each?

21. A farmer has 320 sheep in 2 pastures; and $\frac{1}{2}$ of the number in one pasture increased by $\frac{1}{10}$ of the number is the number in the other pasture; how many sheep are in each pasture?

22. What number is that of which $\frac{2}{3}$ exceeds $\frac{2}{5}$ by 8?

23. If a certain number be increased by 10, the sum is $\frac{7}{5}$ of the number; what is the number?

24. When Hiram was 8 years younger, his age was $\frac{3}{5}$ of his present age; how old is he?

25. A purse and its contents are worth \$1.50, but if you take out 25 cents, $\frac{1}{3}$ of the remaining money is $\frac{1}{2}$ of the value of the purse; what is the value of the purse?

LESSON IX.

1. One number is 3 times another, but if 10 be added to each, the first will become twice the second; what are the numbers?

ANALYSIS.—Since the first is 3 times the second, 10 added to 3 times the second makes it twice the sum of 10 and the second, that is, 20 more than twice the second. If 10 more than 3 times the second is 20 more than twice the second, then 3 times the second must be $20 - 10 = 10$ more than twice the second. Therefore the second number must be 10, and the first number 30.

2. One number is 3 times another, but if 8 be added to each, the first becomes twice the second; what are the numbers?

3. One number is 4 times another, but if 10 be added to each, the first becomes 3 times the second; what are the numbers?

4. A father is 5 times as old as his son, but in 8 years he will be 3 times as old; how old is each?

5. A had 5 times as much money as B, but after each had made \$1500, A had twice as much as B; how much money had each?

6. C had $\frac{1}{6}$ as many acres of land as D, but after each had bought 24 acres more, D had twice as many as C; how many acres had each?

7. Said John to Mary, "I am twice as old as you, but 10 years ago I was 4 times as old as you." How old is each?

8. E had 7 times as many sheep as F, but when each had lost 5 sheep, E had 9 times as many as F; how many had each?

9. Thirty years ago my grandfather was 11 times as old as I was, and 10 years ago he was only 3 times as old as I was; what is the age of both?

10. A and B were partners; A furnished $\frac{3}{4}$ of the capital, and B $\frac{1}{4}$ and his labor; after they had shared equally a gain of \$4000, and put it in as capital, A had $\frac{2}{3}$ of the capital; how much had each?

11. Alfred bought some oranges at 2 cents apiece, and as many more for 3 cents apiece, and sold them at the average rate of 3 for a dime; what was his average gain per orange?

ANALYSIS.—The average cost by *twos* was $2 + 3 = 5$ cents; hence the average cost per orange was $\frac{1}{2}$ of $5 = 2\frac{1}{2}$ cts. The average selling price was $\frac{1}{3}$ of $10 = 3\frac{1}{3}$ cts.; hence the average gain was $3\frac{1}{3} - 2\frac{1}{2} = \frac{5}{6}$ of a cent.

12. Frederick bought some melons at 3 cents each, and as many more at 5 cents each, and sold them so as to average 10 cents for 2; what was the gain per melon?

13. A speculator bought some potatoes for 20 cts. a bu., and twice as many more for 32 cts. a bu., and sold them at 50 cts. a bu.; what was the average gain?

14. A news-boy bought the Courier at $\frac{3}{4}$ of a cent apiece, and an equal number each of the Gazette, Journal, and Ledger at $1\frac{1}{4}$ cents apiece, and sold them at the average rate of 4 for a dime; what was the average gain per paper, and how much would he make on a dozen of each?

15. A merchant mixed some tea at 40 cts. a pound with 4 times as much at 60 cts. a pound, and sold the mixture at 75 cts. a pound, thereby gaining \$19; how many pounds of each kind of tea were there?

16. A flour speculator bought a lot of flour at \$3 a barrel, and 5 times as much at \$4 a barrel; but he was compelled to sell it at \$3 $\frac{1}{2}$ a barrel, thereby losing \$60; how much flour of each kind had he?

17. A merchant mixed sugar at 9 cents a pound with 6 times as much at 9 $\frac{1}{2}$ cents a pound, and sold the mixture at 10 cts. a pound, gaining \$8; how much sugar of each kind was in the mixture?

18. A huckster bought some apples at the rate of 2 for a cent, and as many more at the rate of 3 for a cent. He sold them together at the rate of 5 for 2 cents, and lost 4 cents by the operation. How many of each kind did he buy?

19. If I gain $\frac{4}{5}$ of a cent on every peach sold, how many must I sell to gain 80 cents?

LESSON X.

1. A and B are walking in the same direction, A being 90 of his own steps ahead of B; B's step is $\frac{5}{4}$ of A's, and A takes 4 steps while B takes 5; in how many of B's steps will B overtake A?

ANALYSIS.—Since 1 of B's steps is $\frac{5}{4}$ of 1 of A's, 5 of B's steps will be 5 times $\frac{5}{4} = \frac{25}{4} = 6\frac{1}{4}$ of A's steps. Since A only takes 4 of his own steps while B takes $6\frac{1}{4}$ of A's steps, B gains on A $2\frac{1}{4}$ of A's steps for every 5 of B's steps; therefore to gain 90 of A's steps B must take as many times 5 of his own as $2\frac{1}{4}$ is contained times in 90; since $2\frac{1}{4}$ is contained 40 times in 90, B must take $40 \times 5 = 200$ paces to overtake A.

2. A fox is 80 leaps ahead of a hound, and takes 3 leaps while the hound takes 2, but 1 of the hound's is equal to 2 of the fox's; in how many leaps will the hound catch the fox?

3. At a quarter to 4 o'clock, how many minutes before the minute-hand overtakes the hour-hand?

4. C is 40 steps ahead of D, and takes 4 steps as often as D takes 3; but 1 of D's steps is equal to 2 of C's; in how many steps will D overtake C?

5. A hare is 50 leaps ahead of a dog, and takes 5 leaps while the dog takes 2; but 5 of the dog's leaps are equal to 15 of the hare's; how many leaps must the dog make to overtake the hare?

6. At half-past 4 o'clock how many minutes will it be before the minute-hand will be opposite the hour-hand?

7. E in walking 90 rods overtook F; and $\frac{4}{5}$ of the distance walked by F was the distance between them when E started; what was the distance?

8. If A in taking 60 steps overtakes B, how many steps ahead was B, if 6 of A's equal 5 of B's, and A takes 5 while B takes 3?

9. M, having 25 steps the start of N, takes 3 steps while N takes 6, but 2 of M's steps equal 3 of N's; how many steps will each take before N overtakes M?

10. A steamboat, whose speed in still water is $10\frac{1}{2}$ miles an hour, plies between two cities on a river whose velocity averages $3\frac{1}{2}$ miles an hour. If the trip down and up takes 9 hours, what are the distance between the places, and the times of the trips up and down?

ANALYSIS.—Going down, the boat's speed is $10\frac{1}{2} + 3\frac{1}{2} = 14$ miles per hour; going up, her speed is $10\frac{1}{2} - 3\frac{1}{2} = 7$ miles per hour. Therefore her time of travel, 9 hours, must be divided into two parts which are to each other as 14 to 7, or as 2 to 1. Hence $\frac{2}{3}$ of 9 hours, which is 6 hours, is the time of the trip down, and $\frac{1}{3}$ of 9 hours, which is 3 hours, is the time of the trip up. Hence 6 times 7 miles, or 3 times 14 miles, or 42 miles, is the distance between the cities.

11. A boat, whose speed in still water is 12 miles an hour, takes 12 hours for the round trip on a river whose velocity is 4 miles an hour; how far does she go?

12. A boat, whose speed is 15 miles an hour in still water, takes 10 hours to go and come between two places on a river whose current averages 3 miles an hour; what is the distance between the places?

13. How far can a person ride out at the rate of 7 miles an hour, to walk back at the rate of 3 miles an hour and get back in 1 hour from starting?

14. How far can a person walk out at the rate of 4 miles an hour, to ride back at the rate of 8 miles an hour, and get back in $1\frac{1}{2}$ hours from starting?

15. If a boat, whose speed is 15 miles an hour in still water, is 3 hours going a certain trip down stream and 6 hours in returning, what is the velocity of the stream and the length of the trip?

16. If a boat, whose speed is 15 miles an hour in still water, is 2 hours performing a certain trip down stream and 3 hours in returning, what is the velocity of the stream and the length of the trip?

17. If two boats start from cities 91 miles apart on a stream whose velocity is 4 miles an hour, the boat going up having a rate of 16 miles an hour in still water, the other 12 miles an hour in still water, at what point in the trip will they meet, and how soon?

LESSON XI.

1. If we consider 64 as produced by 8 times 8, what is 64 called in reference to 8?

ANS. 64 is called the *square* of 8.

2. What is the square of a number?

ANS. The square of a number is the product produced by multiplying the number by itself.

3. If we consider 64 as produced by 4 times 4 times 4, what is 64 called in reference to 4?

ANS. 64 is called the *cube* of 4.

4. What is the cube of a number?

ANS. The cube of a number is the product produced by multiplying the square of the number by the number. Therefore in producing the cube of a number, that number is used three times as a factor.

5. If 64 is the square of 8, what is 8 called in reference to 64?

ANS. 8 is called the *square root* of 64.

6. If 64 is the cube of 4, what is 4 called in reference to 64?

ANS. 4 is called the *cube root* of 64.

7. What is the square of 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 20? 30? 40? 50? 60? 70? 80?

MODEL OF ANSWER.—Because $2 \times 2 = 4$, the square of 2 is 4.

8. What is the cube of 2? 3? 4? 5? 6? 7? 8? &c.

MODEL OF ANSWER.—Because $2 \times 2 \times 2 = 8$, the cube of 2 is 8.

9. What is the square of $\frac{1}{2}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{6}{7}$? $\frac{7}{8}$? $\frac{8}{9}$? $\frac{9}{10}$? $\frac{10}{11}$? $\frac{11}{12}$?

10. What is the cube of $\frac{1}{2}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{6}{7}$? $\frac{7}{8}$? &c.

11. What is the square root of 1? 4? 9? 16? 25? 36? 49? 64? 81? 100? 121? 144? 400? 900?

MODEL OF ANSWER.—Because $2 \times 2 = 4$, the square root of 4 is 2

12. What is the cube root of 1? 8? 27? 64? 125? 216? 343? 512? 729? 1000? 1321? 1728?

MODEL OF ANSWER.—Because $2 \times 2 \times 2 = 8$, the cube root of 8 is 2.

13. What is the square root of $\frac{1}{4}$? $\frac{4}{9}$? $\frac{16}{25}$? $\frac{49}{64}$?

14. What is the cube root of $\frac{1}{8}$? $\frac{27}{64}$? $\frac{125}{216}$? $\frac{343}{512}$?
 $\frac{729}{1000}$? $\frac{1321}{1728}$?

15. Wendell, being asked his age, said that $\frac{1}{4}$ of 3 times the square of his age was 75 years; how old was he?

ANALYSIS.—If 75 was $\frac{1}{4}$, 4 times 75, or 300, was 3 times the square of his age; and $\frac{1}{3}$ of 300, which is 100, was the square of his age; therefore the square root of 100, or 10, was his age.

16. The square of a number is 121; what is the square of half the number?

17. Half the square of a number is $40\frac{1}{2}$; what is the square of half the number?

18. Twice the square of a number is 200; what is the square of twice the number?

19. Three-fourths of the cube of a number is 48; what is the cube of three-fourths of the number?

20. What is the difference between $\frac{2}{3}$ of a cubic yard, and $\frac{2}{3}$ of a yard cubed?

21. A party composed of twice as many ladies as gentlemen spent \$25, the gentlemen paying all the expense, and each paying as many dollars as there were gentlemen in the party. How many persons of each kind were in the party?

22. How many times the square of a number is the square of twice the number?

23. How many times the cube of a number is the cube of twice the number?

24. How many times the square of a number is the square of three times the number?

25. How many times the cube of a number is the cube of three times the number?

26. What part of the square of a number is the square of $\frac{1}{2}$ of the number? $\frac{1}{3}$ of the number? $\frac{1}{4}$ of the number?

27. What part of the cube of a number is the cube of $\frac{1}{2}$ of the number? $\frac{1}{3}$ of the number? $\frac{1}{4}$ of the number?

28. The square of twice a number is 32 more than twice the square of the number; what is the number?

29. What is the length of a square cellar, 8 ft. deep, whose capacity is 3200 cubic feet?

LESSON XII.

1. The square of 3 times a number is 24 more than 3 times the square of the number; what is the number?

ANALYSIS.—The square of 3 times a number is 9 times the square of the number. If $9 - 3 = 6$ times the square of the number is 24, then $\frac{1}{6}$ of $24 = 4$ must be the square of the number; therefore the number is 2.

2. The square of 3 times a number is 54 more than 3 times the square of the number; what is the number?

3. The square of 4 times a number is 300 more than 4 times the square of the number; what is the number?

4. The square of 5 times a number is 720 more than 5 times the square of the number; what is the number?

5. The cube of twice a number is 48 more than twice the cube of the number; what is the number?

6. The square of $\frac{1}{2}$ of a number is 9 less than $\frac{1}{2}$ of the square of the number; what is the number?

7. The square of $\frac{1}{3}$ of a number is 8 less than $\frac{1}{3}$ of the square of the number; what is the number?

8. One-half of the cube of a number is 24 more than the cube of $\frac{1}{2}$ of the number; what is the number?

9. Five persons hire a conveyance for a fair, but 3 more join them, by which the expense of each of the first parties was diminished \$1.50; what was paid for the conveyance?

FIRST ANALYSIS.—Each of the 5 was to pay $\frac{1}{5}$ of the price, but, after 3 joined them, each paid $\frac{1}{8}$. If $\frac{1}{5} - \frac{1}{8} = \frac{3}{40}$ was \$1.50, then $\frac{1}{5}$ of \$1.50, or 50 cts., was $\frac{1}{40}$, and 40 times 50 cts., or \$20, was the price.

SECOND ANALYSIS.—If 3 men make a diminution of $5 \times \$1.50 = \7.50 , each man paid $\frac{1}{5}$ of \$7.50, or \$2.50, and 8 men paid 8 times \$2.50, which is \$20.

10. Three young men hired a pew, but by taking in 2 more, the expense of each was diminished \$6; what was the rent of the pew?

11. A boy bought 20 apples, but 5 of them being worthless increased the cost of each of the rest $\frac{1}{2}$ of a cent; what was the cost of the 20 apples?

12. Ten men chartered a boat for a trip, but 2 of them failed to pay, by which the expense of each of the rest was increased \$1; what was charged for the boat?

13. John bought 25 pears, and, when Samuel gave him 15 more, he found the average cost of each diminished $\frac{3}{10}$ of a cent; what did the 25 pears cost?

14. A party of 30 gentlemen agreed to bear the expense of a pic-nic, but when 5 of them refused to carry out the arrangement, the expense of each of the rest was increased 20 cents; what was the cost of the pic-nic?

15. I sold goods at 10% gain. Had they cost \$100 more, I should have lost 10% by selling as I did. What did the goods cost?

ANALYSIS.—The goods were sold at $\frac{11}{10}$ of their cost; but if their cost had been \$100 more, $\frac{11}{10}$ of their actual cost would have been $\frac{9}{10}$ of the new cost; therefore $\frac{1}{9}$ of $\frac{11}{10} = \frac{11}{90}$ of cost would be $\frac{1}{10}$ of new cost, and 10 times $\frac{11}{90} = \frac{110}{90} = \frac{11}{9}$ of cost would be new cost. Since \$100 adds $\frac{11}{9} - \frac{9}{9} = \frac{2}{9}$ to cost, $\frac{1}{2}$ of \$100, or \$50, is $\frac{1}{9}$ of cost, and 9 times \$50, or \$450, is cost; therefore the goods cost \$450.

16. Goods were sold at 10% gain. If they had cost \$60 more, at the same selling price there would have been 10% loss; what did the goods cost?

7. A merchant marked broadcloth for selling, at what he thought was 20% gain; but, finding that the cloth had cost \$3 per yard more than he had thought, he saw that he was about to sell it at 25% loss; what did the cloth cost per yard?

18. A merchant sold goods at 20% gain, but, if they had cost \$150 less, his gain at the selling price would have been 30% more; what did the goods cost?

19. A farmer sold his wheat crop at 10% loss, but, if he had raised it \$25 cheaper, he would have gained 20%; what did his crop cost him?

20. A speculator sold flour at 25% loss, but, if the flour had cost him \$1 more per barrel, his loss would have been 40%; what did the flour cost per barrel?

21. I sold goods at 100% gain; had they cost \$50 more, I should have gained $33\frac{1}{3}\%$; what did they cost?

LESSON XIII.

1. If an article had cost me 10% less, my gain would have been 12% more; what was the gain per cent.?

ANALYSIS.—The new cost is 90% of the old. To sell at the old cost would be a profit of $\frac{10}{90}$, or $11\frac{1}{9}\%$, on the new cost, and would be no profit on the old. Since 1% of old cost is $\frac{1}{90}$, or $1\frac{1}{9}\%$, of new cost, every 1% profit on old cost gains $\frac{1}{9}\%$ more on new cost. Hence, to gain 12% — $11\frac{1}{9}\% = \frac{8}{9}\%$ would require as many per cent. profit on the old cost as $\frac{1}{9}$ is contained times in $\frac{8}{9}$. Since $\frac{1}{9}$ is contained in $\frac{8}{9}$ 8 times, the actual profit in the transaction was 8%.

2. If an article had cost 20% less, the gain would have been 30% more; what was the gain per cent.?

3. If cost had been 8% less, gain would have been 10% more; what was the gain per cent.?

4. If cost had been 16% less, gain would have been 20% more; what was the gain per cent.?

5. If cost had been 4% less, gain would have been $4\frac{2}{3}\%$ more; what was the gain per cent.?

6. If cost had been 20% less, gain would have been 28% more; what was the gain per cent.?

7. If cost had been 5% less, gain would have been 6% more; what was the gain per cent.?

8. If cost had been 12% more, gain would have been 15% less; what was the gain per cent.?

ANALYSIS.—The new cost is 112% of the old; 1% of the old cost is $\frac{1}{112}$ of the new; $\frac{1}{112} = \frac{25}{28}\%$. Therefore every $\frac{25}{28}\%$ of the new cost is $\frac{3}{28}\%$ less than 1% in the old; and, to be 15% less, there must be as many per cent. of the old as $\frac{3}{28}$ is contained times in 15, which is 140% of the old; therefore the gain was 40%.

9. If cost had been 10% more, gain would have been 12% less; what was the gain per cent.?

10. If cost had been 20% more, gain would have been 25% less; what was the gain per cent.?

11. Two merchants sell cloth at the same marked price, but one, having given 25% more than the other makes 30% less; what per cent. on cost is the marked price?

12. A man had money in copper, silver, and gold pieces. He had \$6 in silver, and if he put the copper with the silver, it was twice the gold; but if he put the silver with the gold, it was 5 times the copper; how much of each kind of money had he?

ANALYSIS.—Since the copper is $\frac{1}{5}$ of the gold and silver, it is $\frac{1}{5}$ of the gold + $\frac{1}{5}$ of \$6. Since the copper plus \$6 is twice the gold, it follows that $\frac{1}{5}$ of the gold + $\frac{1}{5}$ of \$6 + \$6 equals twice, or $\frac{10}{5}$, of the gold. Therefore $\frac{1}{5}$ of the gold is $\$3\frac{3}{5}$ less than $\frac{10}{5}$ of the gold; that is, $\frac{9}{5}$ of the gold is $\$3\frac{3}{5}$, and $\frac{1}{5}$ of the gold is $\frac{1}{9}$ of $\$3\frac{3}{5} = \$\frac{4}{3}$, and $\frac{5}{9}$, or all the gold, is 5 times $\$4 = \$20 = \$4$; therefore the copper must be $\$6 + \$4 = \$10 \div 5 = \2 , and he had \$2 in copper, \$6 in silver, and \$4 in gold.

NOTE.—Let the learner solve this question by several other methods of analysis.

13. If you add \$3 in copper to some silver, it will equal some gold; but if you add the copper to the gold, it will be twice the silver; how much money of each kind is there?

14. If a chain costing \$20 be put upon a silver watch, the cost of both is $\frac{1}{2}$ the cost of a gold watch; but if the chain be put upon the gold watch, the cost of both is 4 times the cost of the silver watch; what is the cost of each watch?

15. A has \$40; if he gives it to B, B will have as much as C; but if he gives it to C, C will have twice as much as B; how much has each?

16. Of three casks the first contains 12 gallons. The capacity of the first and third is twice that of the second, and the capacity of the second and third is three times that of the first; what is the capacity of each cask?

17. D's farm contains 80 acres. If he sells it to E, E will then have as many acres as F has; but if E sells his farm to F, F will then have 7 times as many acres as D has; how many acres has each?

18. G has \$40, which is $\frac{1}{3}$ of what H and I have, and I has $\frac{1}{7}$ of what G and H have; how much has each?

LESSON XIV.

1. A merchant, on opening store in the morning, first took in as much money as he had at opening, then paid out \$2, then took in as much as he had left, then paid out \$4, and, closing, found that he had \$12. How much did he have on opening the store?

FIRST ANALYSIS.—On taking in as much as he had at first, he had twice as much as at first; then, paying out \$2, he had twice his first money less \$2. Then, taking in as much more, he had 4 times his first money less \$4. Then paying out \$4, he had 4 times his first money less \$8. Now, if \$12 is \$8 less than 4 times his first money, $\$12 + \$8 = \$20$ is 4 times his first money, and $\frac{1}{4}$ of \$20, or \$5, is what he had on opening.

SECOND ANALYSIS.—Since he had \$12 at closing, he had $\$12 + \$4 = \$16$ before the last payment, and $\frac{1}{2}$ of \$16, or \$8, before last sale, and $\$8 + \2 , or \$10, before first payment, and $\frac{1}{2}$ of \$10, or \$5, before first sales. Therefore he had \$5 on opening store.

2. A began the day with taking in as much money as he had, then paid out \$6, then took in as much as he had left, then paid out \$3, and had \$25 left. How much had he at first?

3. B first took in twice as much as he had, and then paid out \$1, then took in three times as much as he had left, then paid out \$5, and had \$15 left. How much had B at first?

4. C first took in $\frac{1}{2}$ as much as he had, then paid out \$3, then took in $\frac{1}{3}$ as much as he had left, then paid out \$2, and had \$10 left. How much had C at first?

5. D first took in $1\frac{1}{2}$ times as much as he had, then paid out \$5, then took in $2\frac{1}{5}$ times what he had left, then paid out \$8, and had \$24 left. How much had D at first?

6. E first paid out $\frac{1}{2}$ of what he had, then took in \$10, then paid out $\frac{1}{4}$ of what he then had, then took in \$8, and found that he had \$20. How much money had E at first and how much did he make that day?

7. Matthew and Mark have the same sum of money, but if Matthew gains \$8, and Mark loses \$4, Matthew will have three times as much as Mark; how much money has each?

ANALYSIS.—After Matthew's gain and Mark's loss, the difference between their sums is $\$8 + \$4 = \$12$. Since Mark has then 3 times as much as Matthew, \$12 must be $\frac{2}{3}$ of Matthew's money, and $\frac{1}{2}$ of \$12, or \$6, must be $\frac{1}{3}$ of it, and 3 times \$6, or \$18, must be Matthew's money, and \$4 must be Mark's money. Therefore each had, at first, $\$18 - \$8 = \$10$.

8. Messrs. Brown and Moore started as partners of equal capital, but when Brown had run in debt \$500, and Moore had saved \$1000, Moore was worth four times as much as Brown. What was their capital?

9. Smith and King invested the same sum in speculation. Smith made \$1500, and King lost \$1500, when Smith's venture was worth 7 times as much as King's. What did they invest?

10. Isaac and Jacob have an equal number of cents, but if Isaac gives Jacob 2 dimes, Isaac will then have $\frac{1}{9}$ as much money as Jacob. How much has each?

11. When A and B met, each had the same sum of money, but when A had paid B a debt of \$3, B had four times as much as A. How much had each?

12. If, 3 hours ago, the time to midnight was 3 times what it will be 3 hours hence, what o'clock is it?

13. If, 20 years ago, my age was $\frac{1}{4}$ of what it will be 10 years hence, how old am I?

14. If, 4 years ago, $\frac{3}{5}$ of A's age was $\frac{3}{7}$ of what it will be 4 years hence, what is his age?

15. B made in speculation \$3000, and then found that $\frac{2}{3}$ of his money was $\frac{5}{6}$ of what he had at first. How much money had he?

16. C has twice as much money as D, but if C gives D \$6, they will have the same sum. How much has each?

17. E has 3 times as much money as F; but if E gives F \$8, F will have $\frac{2}{5}$ as much as E. How much has each?

LESSON XV.

1. How much water must be added to alcohol which is 80% strong, to make the mixture 60% strong?

ANALYSIS.—If 60% of the new mixture equals 80% of the old, then 1% of the new is $\frac{1}{60}$ of 80% = $\frac{80}{60}\%$, or $\frac{4}{3}\%$, of the old, and 100% of the new is 100 times $\frac{4}{3}\%$ of the old, that is, $4\frac{2}{3}\%$, or $133\frac{1}{3}\%$, of the old. Therefore the water to be added is $133\frac{1}{3} - 100\% = 33\frac{1}{3}\%$, or $\frac{1}{3}$, of the old mixture.

2. How much water, added to alcohol 96% strong, will reduce it to 80% strong?

3. How much water, added to 25 gallons of alcohol 90% strong, will make it 75% strong?

4. How much alloy, added to 9 oz. of gold $2\frac{2}{4}$ or 22 carats fine, will make it $1\frac{8}{4}$ or 18 carats fine?

5. How much alloy, added to 6 oz. of gold 20 carats fine, will make it 15 carats fine?

6. How much weight of fresh water, added to 50 lbs. of brine 4% strong, will make it 3% strong?

7. A chemist dissolved 3 oz. of a salt in 27 oz. of water; but when his assistant had by mistake added more water, he, to find the quantity, evaporated 1 oz. of the new mixture, and found $\frac{1}{5}$ of an oz. of the salt; how much water had been added by the assistant?

8. If 5 pwt. of gold, 22 carats fine, be melted with 3 pwt., 14 carats fine, how fine is the mixture?

ANALYSIS.—If each of 5 pwt. contain $2\frac{2}{4}$ pwt. pure gold, all contain 5 times $2\frac{2}{4}$, or $11\frac{0}{4}$, pwt. pure gold. If each of 3 pwt. contain $1\frac{4}{4}$ pwt. pure gold, all contain 3 times $1\frac{4}{4} = 4\frac{2}{4}$ pwt. pure gold. Therefore the mixture, 8 pwt., contains $11\frac{0}{4} + 4\frac{2}{4} = 15\frac{2}{4}$ pure gold, and 1 pwt. of the mixture contains $\frac{1}{8}$ of $15\frac{2}{4} = 1\frac{9}{4}$ pure gold. Therefore the mixture is 19 carats fine.

9. If 8 oz. of gold, 18 carats fine, be melted with 4 oz., 15 carats fine, how fine is the mixture?

10. If 10 oz. of gold, 20 carats fine, be melted with 5 oz., 14 carats fine, how fine is the mixture?

11. If $4\frac{1}{2}$ pwt. of gold, 18 carats fine, be melted with $3\frac{1}{2}$ pwt., 22 carats fine, how fine is the mixture?

12. If 1 gallon of alcohol, 96% strong be mixed, with 1 quart, 81% strong, how strong is the mixture?

13. How much alcohol, 90% strong, must be mixed with 4 gallons, 75% strong, to make the mixture 85% strong?

ANALYSIS.—Since the 90% is 5% stronger than the mixture, and the 75% is 10% weaker than the mixture, it will take $\frac{1}{2}$, or 2 times as much of the 90% as of the 75%, to raise the mixture to 85%; 2 times 4 are 8; therefore it will take 8 gallons of 90%.

OR, Since the 90% is 5% stronger than the mixture, and the 75% is 10% weaker than the mixture, it will take $\frac{5}{10}$, or $\frac{1}{2}$, as much of the 75% as of the 90%, to reduce the 90% to 85%. If 4 gallons are $\frac{1}{2}$, 8 gallons must be the quantity of 90% needed.

14. How much alcohol, 96% strong, must be mixed with 8 gallons, 60% strong, to make the mixture 80%?

15. How much alcohol, 70% strong, must be mixed with 9 gallons, 90% strong, to make the mixture 85%?

16. If I buy 12 shares of railroad stock at \$40, and the price falls to \$20, how many shares must I then buy to sell without loss, when the price rises to \$32?

17. If I buy 100 bushels of wheat at 50 cts. a bushel, how many could I buy at \$1 a bushel, so as to lose nothing if I have to sell at 90 cents a bushel?

18. How many pwt. of gold, 15 carats fine, must be melted with 15 pwt., 22 carats fine, to make the mixture 18 carats fine?

19. How many oz. of gold, 21 carats fine, must be melted with 4 oz. of 15 carats, and 2 oz. of 18 carats, to make the mixture 19 carats fine?

PROMISCUOUS EXERCISES.

1. A had a certain sum of money, and when he had received \$36, and spent $\frac{3}{4}$ of what he then had, he found that the remainder was $2\frac{1}{2}$ times what he had at first; how much had he at first?

2. After B had spent \$12, and earned $\frac{1}{9}$ of what he then had, he had $\frac{2}{3}$ of what he had at first; how much had he at first?

3. After C had spent $\frac{1}{3}$ of his money, and then earned \$5, he had $\frac{3}{4}$ of what he had at first; how much had he at first?

4. D and E, partners, lost \$2000, then made 50% on their remaining capital, when they found that they had on the whole made 20% on their first capital; what was the capital of each, if D's was to E's as 2 to 3?

5. A and B bought a melon for 24 cents, A giving 15 cents and B 9 cents. C offered them 8 cents to divide equally, to which they agreed. How should the 8 cents be divided between A and B? ANS. A 7 cts., B 1 ct.

6. D had 2 shares and E 4 shares in the firm of D and E. They agree to admit F and make all their portions equal, on F's paying them \$2000. How should F's money be shared by D and E?

7. M furnished 1 loaf, and N 5 loaves, for supper; they admit O and share equally, on his paying 20 cents; how should the account be adjusted?

8. If John can hoe a patch of corn in $\frac{7}{8}$ of an hour, and James in $\frac{3}{4}$ of an hour, how long would it take both to finish it after John has hoed $\frac{1}{2}$ of an hour?

9. A, B, C, D, E and F made up a subscription; A subscribed \$10, which was $\frac{1}{5}$ of the joint subscription of

B and C; C's was $\frac{1}{4}$ of that of A and B; and if 20 times the united subscription of A, B, and C were divided into parts which are to each other as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$, those parts would be respectively 10 times D's subscription, $12\frac{1}{2}$ times E's, and $16\frac{2}{3}$ times F's. What was given by each?

10. Robert, Samuel and Thomas had in all \$1.50; Samuel had 5 cents more than Robert, and Thomas had 20 cents more than Robert and Samuel together; how many cents had each?

11. C and D have the same annual income; C saves $\frac{1}{4}$ of his, and D, who spends \$300 annually more than C, thereby runs in debt \$300 in 6 years; what is the annual income of each?

12. A bought a watch, and then sold it for \$35, thereby losing $12\frac{1}{2}\%$; he then bought another and sold it at 10% gain, and found that he had finally neither gained nor lost; what did he give for each watch?

13. The toll on 2 loads of oats, one of 150 bushels, the other 240, was in proportion to the number of bushels in each load. On the first it was 15 cents less than the price of 5 bushels, and on the second it was 45 cents more than the price of 5 bushels; what was the price of the oats per bushel?

14. A, B and C were partners. A's stock gained \$120 in 3 months, B's \$150 in $2\frac{1}{2}$ months, and C's \$210 in $4\frac{1}{2}$ months. C's stock was \$350; what was the whole stock?

15. A merchant marked his goods $33\frac{1}{3}\%$ above cost, and then sold at 10% discount on marked price; what per cent. was his profit? What per cent. profit would the buyer make, if he resold them at marked price?

16. David has 12 cents, and George has as many as David, and $\frac{1}{4}$ of what both have, beside; how many has George?

17. Said A to B, "Give me \$10, and I shall have as much money as you." B replied, "Give me \$8, and I shall have twice as much as you." How much had each? Has this question more than one answer? Why?

18. C and D started on a journey with the same sum

of money, but after C had spent \$10 and D \$15, $\frac{1}{4}$ of what C had left was $\frac{1}{3}$ of what D had left. What sum of money had each at first?

19. A, B and C have to pay a debt of \$150; the money of A and B is \$40 less than enough, that of B and C is \$20 less than enough, and that of A and C is \$30 less than enough; how much money has each?

20. After F had spent \$30 more than $\frac{1}{5}$ of his money, he had \$10 more than $\frac{2}{3}$ of it; how much had he at first?

21. One kind of cloth is worth \$1.50 per yd. more than another, and 10 yds. of the second are worth as much as 7 of the first; what is the value of each per yd.?

22. Distribute \$10.30 among A, B, C and D, so that A shall have as many times 4 cents as B has 5 cents, and C shall have as many times 6 cents as A has 3 cents, and D shall have as many times 7 cents as B has 4 cents.

23. The sum of the ages of A, B and C is 80 years, and B's age is $1\frac{1}{2}$ times A's age, and C's age is $2\frac{1}{3}$ times the ages of both; what is the age of each?

24. A boy spent $\frac{1}{2}$ of his money, and then earned 10 cents; he then spent $\frac{1}{3}$ of what he had, and then earned 2 cents; he then spent $\frac{1}{2}$ of what he had, and then earned 1 cent; he then had 12 cents; how many had he at first?

25. If a boy buys apples at the rate of 2 for a cent, and $1\frac{1}{2}$ times as many at the rate of 3 for a cent, and sells them at the rate of 5 for 2 cents, does he gain or lose?

26. A farmer let the hoeing of a field to A for \$36. A alone could hoe it in 24 days, but, after he had worked 4 days, he hired B, who, alone, could hoe the field in 18 days; and, after they two had worked 3 days more, he hired C, who, alone, could hoe the field in 12 days, and they three finished the job; how much should each receive, if each is paid according to his work?

27. A mass of 120 bushels consists of corn and rye in the ratio of 7 to 3. How much rye must be taken away, that the corn may be to the rye as 14 to 4? How much corn must be added to the first mass to make the ratio of the corn to the rye as 11 to 4?

28. Divide 60 into two such parts that the quotient of the first divided by 6 and that of the second divided by 4 will amount to 12.

29. If 30 % of a note which I take for goods is gain, and I sell the note at 16 % discount, what per cent. do I gain?

30. A, B and C buy a 21-acre lot of grass, A paying for 10 acres, B for 8, and C for the rest; into the lot they turn each the same number of cattle, and C pays \$24 to A and B; how should they share the money?

31. A country merchant bought goods at 3 stores in New York, the cash values being to each other as $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$, but, taking them at 6 months' credit on interest at 5 % for 6 months, all his purchases amounted to \$4830; what was the cash value of each purchase

32. A farmer bought a span of horses and a yoke of oxen, at prices which were to each other as $2\frac{1}{3}$ to $1\frac{1}{2}$, and he gave his note for both at 6 % interest. He paid at the end of 5 years 6 months \$75.90 interest. What were the prices of the horses and the oxen?

33. A merchant bought \$2000 worth of goods on credit of 6 months, but, concluding to pay cash, got them at 5 % discount. Money was then worth 6 % per annum, and he sold them 6 mo. afterwards at 20 % advance on credit price. How much did he make on the whole?

Ans. \$443.

34. A man having money at 8 % interest, was only able, when the price of money fell, to get 5 %, by which his annual income from it was diminished \$168; how much money had he at interest?

35. A man, dying, left \$13000 to be divided between his two sons, 13 and 17 years old respectively, so that at 5 % simple interest the parts would amount to equal sums when they were 21 years old; how much had each son?

36. A gentleman divided \$5870 among his three sons, 11, 13, and 15 years of age respectively, so that their shares would, at 5 % simple interest, amount to the same sum when they were 21 years old; what were the shares?

37. A father divided \$8000 between his son and daughter when the son was 16 years old and the daughter 13, and he invested it in such shares at 10% simple interest that the daughter would, at 21 years of age, receive twice as much as the son at 21 years of age; what was the share of each?

38. A man, dying, made his will as follows: if his heir should be a daughter, his widow should have $\frac{2}{3}$ of his estate; but if his heir should be a son, the widow should have $\frac{1}{3}$ of his estate. His heirs were both a son and daughter, on account of which the widow received in equity \$4800 less than she would have received if there had been only a daughter. What would have been the widow's portion if the heir had been only a son?

ANS. \$4200.

39. If a person's wages are \$5 a day, and his board is \$5 a week, and he makes only \$250 in 12 weeks, how many days does he work, and how many days is he idle? (N.B. Allow 6 days for a working week.)

40. If I ask 25% more for an article than it cost me, but sell it at a discount of 20% of my asking price, do I gain or lose?

41. If A and B can hoe a field in 20 hours, and, after a day's work of 10 hours, C joins them and they three finish it in 8 hours, how long would it have taken C alone to hoe the field?

42. What is the hour, if $\frac{2}{3}$ of the time since 7 $\frac{1}{2}$ o'clock A.M. equals $\frac{1}{2}$ of the time to 9 $\frac{1}{2}$ o'clock P.M.?

43. How much water must be mingled with alcohol 80% strong, to make the mixture 50% strong?

44. How far can a boat whose velocity is 8 miles an hour in still water go up a stream whose rate is 4 miles an hour, so as to return in 8 hours?

45. A man 48 years old has a son $\frac{1}{3}$ as old; how long ago was the father 9 times as old as the son?

46. Sixteen persons agreed to bear the expense of certain improvements; but when 6 of them failed to

perform their promise, the cost to each of the rest was increased \$3; what did each pay?

47. I sold goods at 100% gain; but, if I had given \$15 more for them, my gain would have been only 20%; what did the goods cost?

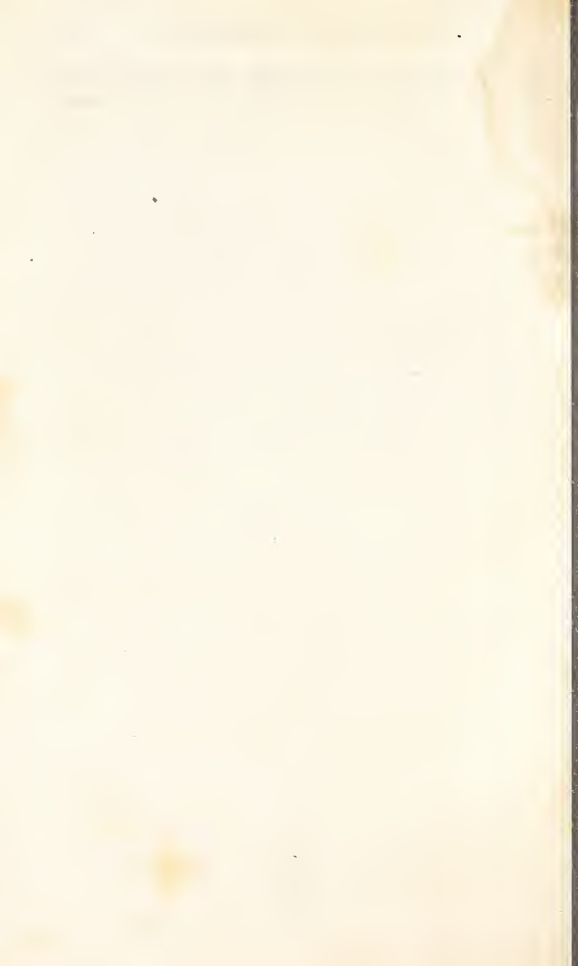
48. If cost had been 100% more, gain would have been 100% less; what was the gain per cent.?

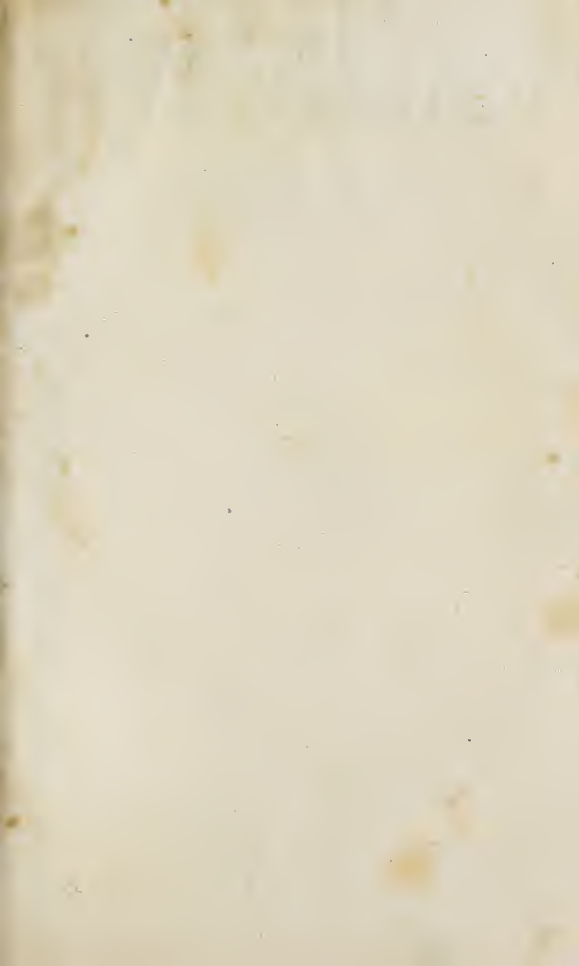
49. If cost had been 50% less, gain would have been 150% more; what was the gain per cent.?

50. If cost had been 20% less, loss would have been 15% less; what was the loss per cent.?

51. If $\frac{1}{10}$ of A's, $\frac{1}{20}$ of B's, and $\frac{1}{5}$ of C's money be put at interest at 10% for 2 years and 6 months, and if the interest thus accruing be put at interest at $\frac{3}{5}$ of the former rate for 4 years 3 months and 18 days, it will amount to \$629. How much money has each, if $2\frac{2}{3}$ of A's part of the principal is $\frac{8}{9}$ of B's, and $\frac{1}{18}$ of B's part of the principal is $\frac{1}{2}$ of C's?

THE END.





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